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RANDOMIZATION PROCEDURES FOR THE ANALYSIS OF EDUCATIONAL EXPERIMENTS.
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CERTAIN SPECIFIC ASPECTS OF HYPOTHESIS TESTS USED FOR ANALYSIS OF RESULTS IN RANDOMIZED EXPERIMENTS WERE STUDIED--(1) THE DEVELOPMENT OF THE THEORETICAL FACTOR, THAT OF PROVIDING INFORMATION ON STATISTICAL TESTS FOR CERTAIN EXPERIMENTAL DESIGNS AND (2) THE DEVELOPMENT OF THE APPLIED ELEMENT, THAT OF SUPPLYING THE EXPERIMENTER WITH MACHINERY FOR ANALYSIS OF HIS RESEARCH. STUDIES OF RANDOMIZATION DESIGN CENTERED ON (1) THE COMPLETELY RANDOMIZED DESIGN, (2) THE RANDOMIZED BLOCK DESIGN, AND (3) THE REPEATED MEASURES DESIGN. THE RESULTS SUGGESTED THAT THE CONVENTIONAL F-TEST WITH A COMPLETELY RANDOMIZED DESIGN IS OPTIMAL AMONG THE THREE PROCEDURES STUDIED FROM THE STANDPOINT OF SIZE AND POWER. THUS, IT WAS DETERMINED THAT THE SIZE AND POWER OF F-TESTS UNDER PERMUTATION PROBABILITY BASES SHOULD AGREE CLOSELY WITH NORMAL THEORY VALUES. (GD)

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RANDOMIZATION PROCEDURES FOR THE ANALYSIS OF EDUCATIONAL EXPERIMENTS

Cooperative Research Project No. 2593 (5-0223)

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1. The basic problem and objectives

The research reported in the present document is directed to certain specific aspects of hypothesis tests used in the analysis of results from randomized experiments. Major concern will reside in the specification of the behavior of particular variance-ratio statistics under a randomization or permutation probability basis rather than the usual sampling probability basis. In all cases except one, the techniques and results which were obtained employ empirical random permutation procedures.

The stimuli which led to a consideration of the problem and to attempts to provide information of use in its solution emerged from the following applied circumstances. Frequently and validly, the point is made by educational and psychological experimenters that the observations encountered in actual research do not comply with the requisites of the usual normal theory hypothesis tests applied in the analysis of experimental results. For this reason, procedures such as analysis of variance hypothesis tests are often viewed as being, more or less, in error. This is to say that these procedures are judged to be inexact, at least with respect to test size, probability of Type I error and power, 1 - probability of Type II error, unless certain assumptions are reasonably well met. As a consequence of this point of view, either less assumptive techniques are sought or the usual techniques are studied in an attempt to discover what effects violations of assumptions have on certain characteristics of the test.

It is these last two items which form the motivation for the present problem. Thus in the primary phase of the problem, interest will be directed to the development of a relatively large scale computer system by means of which unassumptive hypothesis tests on experimental results can be made using analysis of variance derived statistics. The computer system envisaged will be designed to permit randomization tests (or accurate approximations to them) to be run on observations which result from a variety of real or hypothetical experiments. Such a system will be adequate only if it contains an effective input-output component, an efficient random number generator, a fast arithmetic and bookkeeping package, large capacity potential and is written in a language easily adaptable to most large scale computers.

Subsequent to the construction of a flexible computer system, the concern will be with the second aspect of the problem - that of employing this machinery in the study of how probability characteristics of certain normal theory analysis of variance tests are affected by manipulation of the structure of responses hypothesized to have been observed. Since it is the desire to judge test behavior under randomization, relevant descriptive indices of the observations themselves need to be carefully considered, but not supply populations from which they come. These latter investigations will be made on three designs which, because of their utility in applied work, seem worthy of study: the completely randomized design, the randomized block design and the repeated measures design.

In brief summary, the problem contemplated has jointly an applied element, that of supplying the experimenter with machinery for analysis of his research, and a theoretical factor, that of providing information on statistical tests for certain experimental designs.

2. Structure of the report

At this point it will be useful to offer a short description of the organization of the report itself. Specifics concerning the problem and objectives, other than those given in Section I, will be presented throughout the following sections of the report. As is usual, the following section (Section 3) will be dedicated to a review of related literature. Section 4 will present a generalized procedures account of the two facets of the problem, but will leave specific details related to the various designs to be studied in the second facet of the problem to Sections 5, 6 and 7. These sections will relate to the completely randomized, the randomized block, and the repeated measures design, respectively, and each will contain its own specific procedures, results and conclusions. Overall summary and conclusions will be presented in Section 8. In the appendix will be placed tables and those items which are either too lengthy for inclusion in the main body of the report or which have indirect relevance and can be referred to easily without breaking continuity.

3. Related literature and research

Of greatest impact on the present problem, certainly in its primary, definitive stages and to a great degree in the ensuing research, was the work of Wilk and Kempthorne (1956) on the finite model and its use in the conceptualization, representation, and analysis of randomized experiments. Through this work, relatively unknown in education and educational circles, one is led to consider the foundations of the experimental process under randomization--the experimental unit, the randomization pattern or design, possible models useful in the representation of the observational structure of the experiment, and a finite conceptualization dependent on unit and design which provide a basis for obtaining error structures and corresponding inferential statements. Although there is to be found in the Wilk and Kempthorne monograph a good elucidation of the background and a list of references pertaining to randomization theory and its role in experimentation, a most comprehensive treatment of the subject is given by Scheffé (1959). Many of the directly relevant references which Scheffé has included and other work done recently have been summarized below in an attempt to put the present research in reasonable context with its background.

Commencing with Fisher (1925, 1935) the problems of inference in randomized experiments have been studied by many writers. Discussion, controversy and investigation of inferential bases are still going on with great activity, particularly since Bayesian and likelihood function frameworks have been reintroduced and revitalized. Fisher's original writings on design had great impact on statistical inference in many ways, but surely they were basic insofar as they introduced the idea of randomization and the randomization test or, as we shall refer to it hereafter, the permutation test. By means of this procedure it was possible to obtain an exact significance test, although the actual computations were potentially prohibitively laborious. Utilizing the analysis of variance arithmetic scheme and certain variance-ratio functions from it, the permutation test as viewed by Fisher has been used by others to obtain tests with exact size for any set of responses.

Fisher's contention that the validity of normal theory significance tests lay in their ability to approximate exact permutation tests was verified empirically for the randomized block design by Eden and Yates (1933). They showed, in fact, that for real data the size of the "no treatments" F-test under normal theory agreed well with that under permutation for the design studied. Although Fisher was the pioneer on experimental design and randomization; to Neyman, et al (1935) must be attributed the first mathematical treatment of the interrelationship of experimental unit, true yield and design. Pitman (1937a, 1937b) and Welch (1937) were stimulated by the work of Fisher, Eden and Yates, and Neyman to obtain analytical results on the permutation distribution of variance-ratios (actually they used monotonic functions of them) and to show that, with certain regularity conditions placed on the observations, the first four moments of the permutation distributions of certain variance-ratios were equivalent to comparable moments of appropriately defined F-distributions. The conditions on the observations specified by Pitman and Welch were defined differentially for the completely randomized design and for the randomized block design and will be specifically recalled when required in future sections.

Following these foundation works one finds analytical papers by Wald and Wolfowitz (1944) and Silvey (1954) attesting to the fact that the asymptotic distribution of the variance-ratio statistic under permutation was indeed the F-distribution underlying normal theory analyses of variance tests. Moreover, Hoeffding (1952) showed that, in the randomized block design, the F-test under permutation was asymptotically equal in power to the normal theory F-test for fixed test size.

In addition to these analytical results, one can cite several empirical papers which were devoted to studying the distributions of variance-ratio statistics of experimental designs under permutation for null and non-null treatments. In an empirical investigation of the two factor, completely randomized design, with one observation per cell, Hack (1958) found that, except in the case of extremely leptokurtic sample observations, the size of the F-test under permutation was approximately equal to that of the normal theory F-test. In a commentary on the Hack paper, Johnson (1958) adapted Welch's (1937) expressions in order to correct the degrees of freedom of the F-distribution used in the completely randomized design so that the variance-ratio distributions under permutation could be more exactly approximated. Johnson's corrections displayed that for this design leptokurtosis of the total sample had a definite effect on the size of the F-test under permutation. An empirical study by Collier and Baker (1963) revealed that, in the split-plot design, null distributions of variance-ratios under permutation for four data sets were approximately F-distributed.

While a fair number of empirical studies have been completed on the size of the F-test under permutation, little has been done in investigating the power of the F-test under permutation aside from the work reported by Kempthorne, et al (1961). These authors investigated the power of the F-test under the totality of permutations for the randomized block with two treatments on various data sets. Their results indicated good agreement of the power of the F-test under permutation with power of the test under normal theory for the same test size. Similar power findings for the 3 x 8 and the 3 x 15 randomized block design were also reported by Collier and Baker (1963).

To obtain results on power and size under permutation, and also to provide a vehicle for limited applied analysis, a Monte Carlo program was written by Baker and Collier (1961). The program has been used by these authors in the research previously reported and, although not applicable to a wide class of designs, it was valuable as a pilot device for the present study.

The foregoing references point directly to the desirability of inspecting the two prongs of the problem outlined in Section 1. First, there is a definite need for a fast, distribution-free system which can be used in the analysis of experimental results. Such a system, based on permutation arguments can, by employing computer facilities, be made to provide a general purpose, large scale capability.

In addition, there appear to be many gaps in the knowledge basis concerning how permutation procedures, as applied to the analysis of variance, are affected by nonregular patterns of data so as to become divergent from normal theory. Although the literature seems to be firm in asserting that there is good agreement between permutation and normal theory procedures, it will be informative to inspect more thoroughly the designs and circumstances in which this agreement does or does not hold.

4. General Procedures

4.1 Background

The following definitive background is considered necessary to set the basis upon which the permutation computer system was built and on which later empirical runs were made for the three designs investigated. It will be sufficient to use a specific design, say the one factor completely randomized design, in describing the decision procedure of the permutation test and also in defining probability statements to be used later. Extentions of these remarks to other designs are completely straightforward.

Suppose one contemplates observations resulting from a one factor, completely randomized design with I treatments and J observations per treatment. In the case of applications to actual research, the IJ observations to be analyzed will be real responses. However, in studies on the behavior of tests under permutation, the observations are referred to as basal responses, measurements which might have resulted under null treatments, and are chosen with certain desirable attributes. The randomization process which is basic to this design is that for IJ independent experimental subjects J different subjects are assigned to a given treatment at random. Thus a given one of the I treatment levels is administered to J subjects, such that for each subject an experimental response is taken under a single treatment only. Now if treatment effects are assumed to be null, i.e., if the effect is constant over treatments, the IJ responses on hand can be conceived of as being one random pattern out of a set of $(IJ)!/(J!)^I$ possible such patterns. For each of these patterns (permutations) of responses, a statistic is computed which measures average treatment differential, such as the usual F_t = MS(treatments)/ MS(observations within treatment). A distribution of F_t, constructed over the totality of possible permutations, can then be employed to specify a rejection rule with certain test size α^*, say $F_t > F_t^*$, so that $P\left[F_t > F_t^* | T_i = 0\right] = \alpha^*$, where $T_i = 0$ indicates null treatment differences. It is noteworthy that the distribution of F_t under permutation is, in general, a discrete distribution. Hence almost all values of α^* from 0 to 1 are possible only in cases where the number of possible permutations is large. The permutation distribution of F_t depends only on the IJ observations actually on hand and the inference concerning treatment differences assumes only a valid randomization process.

It will be necessary to compare the null distribution of F_t under permutation with that under normal theory. One method by which this can be accomplished is to study the size of the normal theory F-test under permutation. Therefore, it is desirable to specify the probability that F_t is larger than the normal theory rejection point under permutation, i.e., $P\left\{F_t > F_{1-\alpha}\left[I-1, I(J-1)\right] | T_i = 0\right\} = \alpha'$, where $F_{1-\alpha}\left[I-1, I(J-1)\right]$ is the upper α-point on the F-distribution with $I-1$ and $I(J-1)$ degrees of freedom. Reference to previously cited research indicates that the size of the normal theory F-test procedure, using the rejection rule $F_t > F_{1-\alpha}\left[I-1, I(J-1)\right]$ is, for all but extreme conditions, approximately the same under permutation as under normal theory, namely α' is approximately equal to α.

Finally, as has been stated, it will be desirable to inspect the power of the normal theory F-test under a permutation probability basis. For this purpose a similar probability statement as for size is defined, namely $P\left\{F_t > F_{1-\alpha}\left[I-1, I(J-1)\right] | T_i \neq 0\right\} = 1-\beta'$, the probability under permutation of rejecting the hypothesis of no treatment effect, using normal theory decision rules, when in fact there is a non-null additive treatment effect.

Each of these three different probability remarks is capable of being computed, but, because of the large number of possible permutations would be laborious to obtain in practice even with the aid of a computer. A precise estimate of these probabilities can be made by taking a random sample of all possible permutations and, as is to be expected if a sufficiently large sample of permutations is taken, the estimate of a given probability can be made as exactly as desired. (For a thorough treatment of this problem see Dwass (1957).) In the computer system constructed for the present research, random samples were, in fact, taken. Although a more comprehensive description of the computer system which was devised for the project will be given elsewhere, a brief indication of the computer application in connection with the specific completely randomized design will be helpful. For this design the computer made possible the performance of the following tasks, assuming treatment effects to be null: a) real or hypothetical basal responses, after being stored, are randomly permuted according to the design, e.g., in this instance J observations are assigned at random to each of the I treatment level; b) an appropriate variance-ratio, in this case the previously defined F_t, is computed and stored for the original set of responses and for the set in a); c) items a) and b) are repeated a fixed number of times; d) a frequency and cumulative probability distribution with somewhat arbitrary number of class intervals is constructed; e) a cumulative probability distribution is constructed,' using F-distribution points generated by the computer so that cumulative proportions correspond to desired nominal test size values and, finally; f) labeled summaries are printed on output. For specification of the power of the F-test under permutation, the above sequence was altered by inserting additive non-null treatment effects, T_i, between items a) and b) above.

The information generated from the computer makes possible, therefore, the estimation of those probabilities which were originally mentioned in Section 1 and denoted as α^* in this section. Accordingly, a rejection rule for use in applied randomization tests can be constructed. Moreover, comparisons of the size and power of the F-test under permutation and under normal theory can be made, i.e., α and $1-\beta$ can be contrasted with α' and $1-\beta'$, respectively.

4.2 Description of the permutation computer system

The purpose of this program, as has been stated before, was to provide a means by which an investigator could create empirical distributions of the variance ratios for a wide class of experimental designs by means of Monte Carlo techniques. The program allows the user to specify the particular experimental design and the appropriate analysis of variance in a conventional manner using the symbolic degrees of freedom. In addition, the manner in which the observations are to be permuted can also be specified symbolically in a convenient form. The program system will analyze up to 1,000 samples generated by permuting the original data. A frequency distribution is constructed where the class intervals are based on specified points of the appropriate theoretical F-distribution. Various options are available to the user which allow different specifications on the number of class intervals. The output is to the printer.

A general description together with capabilities and limitations of the program is given in the next two subsections. Specific remarks on the use of the system have been included in the information manual of Appendix B together with the program printout in Appendix C.

4.2.1 General Description

The general flow of the program proceeds as described in the paragraph below. The program is read either from magnetic tape or a FORTRAN card deck, into computer memory. The initial subroutine of the program reads in the problem description card which contains a description of the number of F-ratios, the number of error terms, the number of randomization parameter cards, etc. The next subroutine of the program reads in the analysis of variance control cards, one of which describes the dimensionality of the analysis of variance and the otherscontain the control words for the specific analysis of variance. The program then reads in cards describing the error terms and establishes the proper calculations for the error terms desired by the user. The variance ratio calculations are then prestored from information provided on variance ratio description cards. The program then reads in a description of the randomization patterns required to permute the data according to the design of interest and establishes the proper permutation procedures within the computer program. The data and any treatment effect terms are read into computer memory along with a post-processing card which contains information concerning the class intervals of the resulting empirical distributions. Now the program is ready to proceed with analysis.

Initially, the original data are processed through the analysis of variance program without any permutations to provide the error terms and variance ratios for the original data. The second segment of the program will take the original data, permute it according to the randomization patterns, compute the desired variance ratios and build up a frequency distribution for each variance ratio. That is, a counter for the appropriate interval in the frequency distribution is advanced for each variance ratio. This process is repeated until the desired number of samples have been generated and analyzed. This program can analyze up to ten selected variance ratios. The frequency distributions are constructed using theoretical F-points as interval boundaries, twelve F-points being "built-in" with up to eight optional in addition to these twelve.

A further option allows the user to specify that he desires N (where $20 \leq N \leq 50$) class intervals instead of the implicit twelve obtained under standard conditions. If this option is exercised, there may actually be up to N + 3 class intervals since the internal class boundaries are N - 1 equally spaced percentiles and P_{90}, P_{95}, and P_{99}.

4.2.2 Capabilities and limitations of the program

1. Randomization patterns and analysis of variance can be specified for a variable possessing up to 8 subscripts.

2. Only one variable can be employed.
3. Up to 20 stages of randomization can be specified for any given design.
4. All cells of the ANOVA design must contain the same number of observations.
5. The product of the number of levels of the indices must be less than 2500.
6. The number of ANOVA control words cannot exceed 100.
7. The number of variance ratios specified cannot exceed 10.
8. The number of error terms specified cannot exceed 20.
9. The number of sums of squares pooled to form an error term cannot exceed 20.
10. The value -9999.9999 cannot be used as data or result from adding a treatment effect term to data.
11. Embedded blanks are not permitted within any of the control cards.
12. The number of samples generated in a computer run is $\leq$ 1000.
13. All input cards contain a three-character code in columns 1 through 3 to identify their type.
14. A code word for the total sums of squares is not permitted in the ANOVA specification.

5. Permutation analysis of the completely randomized design

Previous research has indicated few results to be available for the completely randomized design, although the design stands as one of the most useful and used designs for research in education and psychology. The empirical results of Hack (1958) and the analytic results of Pitman (1937a) and Johnson (1958) suggest that in this design at least two characteristics of the combined sample, skewness and kurtosis, need be considered in a study of how well the F-test under permutation is approximated by normal theory procedures. Actually, only kurtosis has been shown to be closely involved, although effects of various combinations of skewness and kurtosis have not been extensively inspected. For this reason the investigation in this section will center on manipulating both skewness and kurtosis indices and studying resulting effects on the size and power of the F-test under permutation as compared to nominal size and power of the test under normal theory.

5.1 Procedures

Treatment arrangements selected for this design, judged to offer sufficient generality, were: two single factor layouts--one at two levels and one at three levels; and a pair of two factor, A x B, configurations--a 2 x 2 arrangement and a 3 x 2 setup. The number of observations in each treatment level (or treatment combination in the case of two factors) was in turn six and twelve, resulting in basal response sets with 12, 18, 24 and 36 responses on the one hand and 24, 36, 48 and 72 responses on the other. Selection of skewness and kurtosis levels were based on an earlier study by Box and Andersen (1955), who showed that the size of a corrected F-test agreed with normal theory size for a one factor classification under various combinations of a skewness coefficient, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$ and kurtosis coefficient, $\beta_2 = \frac{\mu_4}{\mu_2^2}$. Following Box and Andersen, combinations of skewness and kurtosis levels computed on the combined sample for a basal response set, were established at $\beta_1 =.0, .5, 1.0$, and $\beta_2 = 2.0, 3.0, 4.0$. Basal response data with selected values of β_1 and β_2 were obtained by means of a computer program which drew samples from normal and non-normal distributions and computed numerical values of these indices. This technique was chosen only to obtain basal responses with certain characteristics, not for the purpose of inferring to these distributions. Since data for the combination $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable in most cases, this combination was deleted from study in all cases. The final panel of combinations of the above β_1, β_2 (with the noted exception) plus two groups of other extreme configurations, $\beta_1 =.0$, $\beta_2 = 5.0$ for all treatment arrangements and $\beta_1 =.0$, $\beta_2 = 10.0$ for the 2 x 12, 3 x 12, 2 x 2 x 6 and 3 x 2 x 6 arrangements. A basal response set was retained here and in the following sections involving β_1 and β_2, only if indices were within ± .10 of nominal values.

It will be recalled for the single factor, completely randomized design with J observations in each of I treatments, that the randomization pattern directs J observations be permuted into each of the I treatment levels. Similarly, the two factor design with I levels of factor A, J levels of factor B and K observations in each of the IJ combinations of the two factors, will employ a permutation process wherein K observations are permuted into each of the IJ treatment combinations. The statistic F_t, previously used in the single factor setup becomes, for the two-factor case, alternately F_A = MS(A)/MS(S(AB)), or F_B = MS(B)/MS(S(AB)) or F_{AB} = MS(AB)/MS(S(AB)), where MS(A) = MS(between levels of A), MS(B) = MS(between levels of B), MS(AB) = MS(interaction A x B) and MS(S(AB))= MS(between responses within A x B cells).

The null distribution of F_t was obtained with 1000 permutations of basal response for each of the 76 combinations of treatment arrangements and skewness and kurtosis levels previously given. In addition each of these configurations was replicated twice in order to discover if results were stable over different data sets. The results have been presented in the form of upper proportions, α', of F_t under permutation corresponding to nominal α of .10, .05, .025 and .01 on the appropriate F-distribution. Thus, for example, in the 2 x 6 case the empirical probability $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = 0]$ of Table 1 is read $P[F_t > F_{1-\alpha}(1,10) | T_i = 0]$ Other probability statements are read in similar fashion throughout the following sections and tables.

In order to obtain results with which the power of the F-test under permutation could be compared with nominal, normal theory power values, additive non-null treatment effects T_i were introduced into the sequence after observations were permuted and before F_t was computed. In this operation the same constant was added to each observation in a treatment or treatment combination,the pattern of constants depending on whether a non-null main effect or interaction was desired. Specification of treatment effects was made through the analysis of variance power charts of Pearson and Hartley (1951). The charts use, as an index of non-nullity, the expression $\phi^2 = \delta^2(n_1 + 1)^{-1}$, where n_1 is the numerator degrees of freedom and δ^2 is a non-centrality factor in the design being considered. In the one-factor, completely randomized design, for instance, $\delta^2 = J\Delta^2/\sigma^2$, where $\Delta^2 = \Sigma(T_i - \bar{T})^2$. Rewriting, $\Delta^2 = \frac{I\phi^2\sigma^2}{J}$, and specifying ϕ^2 and σ^2, one can solve for the value of Δ^2. In the present study ϕ^2 was specified numerically by the power charts, given nominal size α and the desired power, where σ^2 was specified the average of MS(between responses within cells) over 1000 permutations for a given response set. Assuming T_i to have uniform spacing, each T_i can then be specified. Treatment effects for two factor designs were determined in a similar manner although in order to prevent redundancy only non-null A and AB were inspected. In all configurations, treatment effects were defined by fixing the values of ϕ^2 so as to yield nominal power of .60 and .80 for a nominal size of .05. Incidentally, by fixing ϕ^2 in this way, power for nominal size of .01 was obtained, but since this power varied from situation to situation, these quantities are of limited utility. It should be pointed out that all power results refer, whenever there were more than one size replication, to the first replication.

Permutation analysis results are based on 1000 permutations performed by the previously described computer program on a C.D.C. 1604 computer at the Numerical Analysis Center, University of Minnesota.

5.2 Results on test size for the completely randomized design

Tables 1-7 present results on the size of the F-test under permutation. There are several generalizations which seem clear from the tables. First, it is leptokurtosis which affects the size of the tests under permutation, causing it to become smaller than the nominal size, but this effect holds only when the configuration is small and/or the kurtosis level is reasonably high. Thus as the number of observations in the data set becomes larger, the effect of leptokurtosis on test size is lessened, such that only much larger values of β_2 will be deleterious. Reference to Tables 1-7 discloses that only for the small configurations is there a meaningful decrease in test size, e.g., in the 2 x 6 and 3 x 6 sets for $\beta_1 = .0$ and $\beta_2 = 5.0$. A slight effect seems to be apparent in the 2 x 6, $\beta_1 = 1.0$, $\beta_2 = 4.0$ case, but the deviation from the nominal size is not pronounced. Increasing β_2 to 10.0, with $\beta_1 = .0$, in Table 7, by contriving the data sets quite artificially, had greatest effect on test size for the 2 x 12 and 2 x 2 x 6 configurations, although both 3 x 12 and 3 x 2 x 6 sets were affected somewhat. Skewness seemed to be involved only in the 2 x 6, $\beta_1 = 2.0$, $\beta_2 = 4.0$ and then only slightly. No discernible differences were noticed among tests on A, B and AB.

The analytic results of Pitman (1937a) and Johnson (1958) lead one to expect that the effect of leptokurtosis is a reduction in the variability of the distribution of F_t under permutation. This point is, in fact, verified in the present results, for in every deviant case studied the trend is toward a conservative test size. The present results indicate in general that F-test size under permutation agrees well with nominal size if the total possible number of unique permutations is of sufficient size. This is reasonable since the number of unique permutations is an increasing function of the number of observations, but a decreasing function of the number of tied observations (which is increased by larger β_2)

5.3 Results on power for the completely randomized design

The power values of the F-test under permutation, as presented in Tables 8-12, seem to follow a pattern of good agreement with nominal values throughout the cases studied. The most deviant cases are again the smaller configurations, such as the 2 x 6 and the 3 x 6 patterns. Even here the discrepancy between power under permutation and nominal power is about that which one would expect given the previous decrease in test size for these data. Thus we expect and observe more power decrease for $\beta_2 = 5.0$ and $\beta_1 = .0$. However, it would seem that power is less affected than size by manipulation on β_1 and β_2.

5.4 Conclusions

The results of this section are in agreement with those given earlier by Hack (1958) and Johnson (1958) but extend these results by considering both one-way and two-way layouts, by using much fewer observations and by treating power as well as size under permutation. The conclusion is that, for the completely randomized design, the size and power of the F-test under permutation and under normal theory agree exceedingly well. Only for data sets in which the kurtosis level is high and positive is there a noticeable decrease in size and power and this effect is appreciably lessened by increasing sample size. One-way and two-way layouts produced no identifiable differences in results.

The recommendation one can make from the results is that over a reasonably large class of response sets, the F-test under permutation for the completely randomized design agrees with that under normal theory with respect to size and power.

6. Permutation analysis of the randomized block design

The work of Pitman (1937) and Welch (1937) and Scheffé (1959) cited previously has indicated that in the randomized block design the agreement of the F-test under permutation with that based on normal theory is dependent upon the coefficient of variation of the block variances. These authors have shown that the discrete null distribution of the F_t statistic under permutation is approximated by the F-distribution with degrees of freedom adjusted by the factor Q, a measure of heterogeneity of block variances, where $Q = \left[1-\frac{V}{J}\right]^{-1} - \frac{2}{J(I-1)}$ with I = number of treatments, J = number of blocks, $\frac{V}{J} = \frac{1}{J-1}\left[\frac{J\Sigma S_j^2}{(\Sigma S_j)^2} - 1\right]$ and $S_j = \sum_{i=1}^{I} (X_{ij}-\bar{X}_{.j})^2/I$ is the variance of the jth block. The assertion is that for approximate equality of block variances, the value of Q is near unity and the usual F-test is valid, whereas heterogeneous block variances tend to increase the value of Q and, to some degree, invalidate the normal theory F-test.

6.1 Procedures

To obtain a measure of generality for the randomized block design, three basic treatment layouts have been analyzed, namely the three treatment x five block and three treatment x twelve block designs with one observation per cell and the three treatment x three block design with six observations per cell. In an effort to construct data which would introduce some spread on the values of Q, three sets of basal responses were constructed for each of the three treatment x five block and three treatment x twelve block designs. For each design, basal response sets, labeled with the term "variance I", were constructed so that the block variances were constant, whereas data sets identified with the label "variance II" and "variance III" are characterized by increasing values of Q. It will be noted that the spread on Q is greater for the design with five blocks than for the design with twelve blocks. This occurs in spite of the fact that the range of variances is much larger in the twelve block than the five block design and is associated with the well known fact that an increase in the number of blocks, in general, increases the validity of the classical F-test.

For comparative purposes, the values of the five variances for "variance II" data (Q = 1.41) vary, within a multiplicative constant, from 1 to 81, whereas for the three x twelve design the twelve variances in the "variance III" set (Q = 1.43) vary, again within a constant, from 1 to 9528. In each of the two designs, then, some spread was achieved on the index Q, and within each design, the block variances were adjusted so that the average block variance would be the same for three sets of data. In order to achieve some generality, two replications of each data set were obtained for F_t = MS(treatment)/MS(treatment x block interaction) and the results of the runs appear in Table 13.

Size runs were also made for one of the replications in each variance set under three different specifications of a non-null treatment x block interaction effect. Symmetric interactions were constructed and applied to the basal response data as a treatment vector in the power investigations, i.e., added to the basal responses after they had been permuted. In order to obtain some comparability between the 3 x 5 and 3 x 12 designs, the interaction effects were constructed so that the treatment x block mean squares based on the interaction effects alone, were equal (σ^2_{TB}/E(MSTB) = constant) from design to design. The values of σ^2_{TB} for the three treatment x five block interactions were 22.5, 62.5 and 122.5 for specifications 1, 2 and 3, respectively. For the three treatment x twelve block case they were 454.1, 1251.1 and 2490.2 The average null MS(treatments x blocks) for the 3 x 5 and 3 x 12 designs were 105 and 2107.

For the three treatment x three block x six subject design, it was again decided to investigate the effect of differential block variances. Accordingly, three specifications of Q were chosen, namely Q = .67, 1.12 and 3.37, and corresponding data were constructed. However, previous work with the completely randomized design had indicated that the kurtosis within a block might be important here, since, at least for very high, positive kurtosis values, the empirical size under permutation was somewhat deviant from nominal size. Therefore, three specifications of kurtosis of the observations within a block were made, namely β_2 = 3.0, 5.0 and 7.0. Hence taking all combinations of Q and β_2, nine sets of basal responses were obtained and run under null treatments. The empirical F_t-distribution for the treatment (A) effect was obtained using both of the possible error terms, MS(treatment x block interaction) and the MS(between observations within cells). The distribution of F_t for the usual treatment x block interaction (AB) was also obtained under permutation.

Power runs were then made for the three designs with nominal power set at .60 and .80 for $\alpha = .05$. As with the previous designs, treatment effects were calculated by using Hartley and Pearson power charts to obtain ϕ^2, while σ^2 was estimated by using the average on 1000 runs under null treatment effects for that particular error specification.

For the three treatment x three block design with six observations per cell, power was obtained using both error terms employed in the null runs. Tests of the main effect A, using AB interaction as the error term, were separately done for the interaction null (AB) and non-null (AB*) with the treatment effect calculation based on the appropriate size run. As a result for each of the nine sets of base data, power results were obtained for four tests: main effect A tests by AB, A tested by AB*, and A and the AB interaction tested by the within cells error mean square.

6.2 Results on test size for the randomized block design

Size results for the three treatment x five block and three treatment x twelve block randomized design are given in Table 13. These results indicate that, for null treatment x block interaction (the left most column in the table) size under permutation is generally less than nominal size for values of Q larger than unity with the small 3 x 5 design showing the larger discrepancies. In fact, none of the size entries for $Q>1$ were larger than nominal size. For variance I data set, where the block variances are equal, the fact that the value of Q, if generally slightly less than unity, would lead one to expect a slight positive bias in the test. This expectation is in a sense verified, since in Table 13, all entries in the upper left hand cell are larger than the nominal α. Generally, the discrepancies are related to Q. Whereas the results for variance II of 3 x 5 design ($Q = 1.41$) are in fairly good agreement with those for variance set III for the 3 x 12 design ($Q = 1.43$), it is clear that the variance III set for the 3 x 5 design ($Q = 1.81$) yields size results which deviate the most from normal theory.

To interpret the results under a non-null treatment x block interaction it should be recalled that in the randomized block design, the existence of such an interaction tends to inflate the error term and, therefore, decrease the size in comparison to nominal α. The results bear out this fact to some extent in that for variance I set of the 3 x 5 and 3 x 12 designs, all size values decrease as one reads from left to right. A somewhat unusual result, however, is that for both designs combinations of heterogeneous block variances and non-null treatment x block interaction yield size results which are, in certain cases, larger than the results under null treatment x block interaction. For example, size results for the 3 x 5 design with variance II data and treatment x block specification 1 are .074, .037, .024 and .007 for the values of α used, whereas the corresponding values for the same data and null treatment x block interaction are .070, .018, .009 and .000. This tendency is particularly clear for $\alpha \leq .05$.

Table 14 gives size results for the 3 x 3 randomized block design with six observations. From an inspection of columns in the first row it appears that, although for all tests, the empirical size is in good agreement with nominal size when the blocks have homogeneous variance, the test of the AB interaction shows more departure from nominal α than the other two tests. Looking at the table as a whole, there is a general trend towards equality of size results for kurtosis values of 3.0 and 5.0 for tests using S(AB) as error. In addition, the results for kurtosis of 7.0 are, for the most part, somewhat less than those for kurtosis 3.0 and 5.0. The most reasonable explanation is that there is little difference, from the standpoint of experimental size, between data having kurtosis of 3.0 and 5.0, but the kurtosis of 7.0 does slightly lower empirical size. Comparison of results in any column indicate a slight increase in size for tests based on the within cells error and slight positive bias for data with kurtosis values of 3.0 and 5.0 in the variance III set, this bias being largest in the extreme upper tails. On the other hand, test size of the A effect, using the interaction as error is drastically reduced under conditions of heterogeneous block variance. For variance III empirical test size is around .01 to .02 for nominal size of .10, and actually 0.0 for the other values of α considered here.

In summary, very little effect was introduced by changing the within block kurtosis but there is some evidence that exceptionally high values might tend to decrease size for tests based on the S(AB) error mean square. Increasing Q, the index of homogeneity of block variance, tends to slightly liberalize tests based on the within error but greatly reduces the size of the test of A with the interaction as error.

6.3 Results on power for the randomized block design

As can be seen from Table 15, when treatment x block interaction is null, the power of the permutation test for the 3 x 5 and 3 x 12 randomized block designs is very near to the nominal value. Although the size was seen to decrease slightly for increasing variance sets, here it is observed that, generally speaking, the power slightly increases for increasing heterogeneity of block variance.

For increasing treatment x block interaction, Table 15 indicates that empirical power systematically decreases for both designs and all levels of heterogeneity and is markedly less than nominal power in most cases. With regard to the effect of variance heterogeneity, it can be seen that power values decrease with increasing block variability. Thus the unusual facilitative effect of both block heterogeneity and interaction with treatments which was evident in the size runs was not exhibited in the power results. However, there is one peculiarity in Table 15, that of increasing power results for nominal power set at .80 with α = .05 for increasing variance heterogeneity.

Although this trend is only evident at the first two levels of treatment x block interaction, it is consistent there and runs counter to the rest of the power results. With regards to a comparison between the 3 x 5 and 3 x 12 designs, the larger design yields power somewhat nearer nominal for $\alpha = .01$ under null treatment x block interaction and, although the results are far from consistent in the remainder of the table, yields slightly more power than the 3 x 5 design for a majority of the situations investigated.

Although of little practical value, a separate set of power runs were made in which it is assumed that the non-null interaction is anticipated by the experimenter and is taken into account in determining power. These results may be found in Table 16 and will not be discussed further here.

Power results for the 3 x 3 block design with six observations are found in Tables 17 and 18, the two tables being used for nominal power of .60 and .80 respectively. Both tables indicate that there is no systematic effect on power due to changes in block kurtosis values. Considering tests of A using the AB and AB* effects as errors for "variance I" it is clear that these tests are less powerful than the tests based on the within cells error term and are generally less than nominal for the power value defined at $\alpha = .05$.

Whereas the presence of heterogeneous block variances greatly diminished test size for the test using AB as the error term for A, power results indicate otherwise. Both tests show a tendency to yield increasing power as block variances become less homogeneous, the major exception being the test using AB* as error with nominal power of .80 at $\alpha = .05$.

Turning to tests based on the within error term, it is clear that empirical and nominal power are in close agreement throughout the two tables. There is a general decrease in power for the test of AB as block variance becomes more heterogeneous. The difference between variance sets I and III were generally of little practical significance, though, being between .05 and .10. No systematic trends were found when factor A was tested using the S(AB) error term and, in no case, was the intra-block kurtosis found to have any orderly effect on the power results for this design.

6.4 Conclusions

For the randomized block design with one observation per cell the expected decrease in size with increasing block variance heterogeneity was borne out. However, power results did not follow the same pattern, actually showing slightly increasing values. In general, introduction of non-null treatment x block interactions reduced both size and power with the exception that these interactions coupled with heterogeneous block variances yielded some surprisingly large size results. Since the values of Q employed here were rather large it is clear that data which meet the assumption of homogeneity moderately well and which do not suffer from a non-null treatment x block interaction will yield valid results, from a standpoint of both size and power, when analyzed according to the usual analysis of variance procedures.

For the design with six observations per cell the tests based on the within cells error exhibited size and power results close to nominal values and were relatively unaffected by increases in heterogeneity of block variances and kurtosis. The size of the test of the main effect A using the AB interaction as error was greatly reduced under block variance heterogeneity and the power was generally slightly less than that associated with the similar test using the within cells error terms. Whenever the experiment permits its use, the within cells error term should be taken as the error in the randomized block design with more than one observation per cell. If this is permissible, the results here indicate that ordinary procedures will be valid,almost independent, of the type of data being analyzed.

7. Analysis of the repeated measures design

Although the repeated measures design has been utilized with high frequency in educational and psychological research, much controversy has raged over the appropriateness of tests of hypothesis used in its connection. For a good treatment of some of the problems see Lana and Lubin (1963). In view of these discussions it seemed important to supply empirical results related to hypothesis tests used in this design.

The repeated measures structure of main interest in these sections is typical of designs useful in research on learning, verbal association, etc. The design is a two-way, A x B, configuration with I levels of dimension A, J levels of dimension B and with K subjects in each level of A. IK subjects are assigned to levels of A (randomly assigned if A represents a treatment factor), K different subjects to each of the I levels of A. Thereafter, each subject is observed under all levels of factor B. If factor B is a treatment factor, the levels of B are administered randomly over time. If factor B is a non-treatment factor, such as "trials over time", the observations under levels of B are taken non-randomly over time. In any event, each subject is observed J times, i.e., under one level of A in conjunction with all levels (randomly assigned or not) of B. Whether A and/or B are treatment factors or not, an analysis can be performed using sampling processes. However, if A is a treatment factor permutation analysis is possible. In either case, the separate analysis of A, i.e., testing hypotheses on A means, is simply equivalent to tests in the one factor, completely randomized design using subject totals as in basic observations. Only if B is a treatment factor (and an active randomization process is associated with assignment of B levels) is permutation analysis taken to be appropriate.

For this reason, in the present work analyses of results from the design have been approached from two different viewpoints, applicable to the different experimental situations. Thus in Section 7.1 where B is a non-treatment, status dimension, sampling bases (the sole non-permutation investigation made) have been utilized, whereas in Section 7.2, where A and B are treatment factors, the permutation approach has been employed.

7.1 Sampling analysis of the repeated measures design

Among the contributions in the area of repeated measures designs has been the work of Greenhouse and Geisser (1959), who studied the situation in which subjects, divided into independent groups along a classification or treatment dimension, were observed at various times on the same variate or on different variates. These authors pointed to existing multivariate tests and presented conservative tests, utilizing variance-ratios, which were not dependent on analysis of variance assumptions of equal variances of the responses and equal covariances between pairs of responses taken on the same subjects.

Stimulated by this work and that of Box (1954), on whose results Greenhouse and Geisser's conservative test is based, the desire in the present section has been to obtain some selected numerical Monte Carlo sampling results which will be of use in interpreting results obtained in repeated measures analyses. More precisely, the empirical null and non-null sampling distributions of certain variance ratios have been obtained, making possible estimation of the size and the power for a) the conventional F-test; b) a modified F-test due, basically, to Box (1954) and; c) the conservative test of Greenhouse and Geisser. This has been done for several variance and covariance patterns some of which violate the homogeneity assumptions and in a design where permutation analysis is considered to be inappropriate.

7.1.1 Background

The particular layout considered in this section takes the following pattern. Let A represent a classification dimension, the levels of which are denoted by the subscript, $i = 1,2...,I$. Let S refer to a subject dimension, defined within levels of A, members of which are subscripted by $k = 1,2...,K$. Classify IK subjects along A so that there are K different subjects in each level of A. Assume a response is taken on each subject under all levels, $j = 1,2..., J$, of dimension B ("trials", profile levels", etc.). Thus for each of the K subjects in each of the I levels of A, there is a vector of J possible dependent responses on B.

In order to indicate the manner in which the usual covariance assumptions were altered to yield heterogeneous variances and covariances, two conventional models have been given below. Following the two models are presented the covariance specifications which were used in the present study and which follow as a consequence of these models.

Consider first, a linear model often written for the above layout:

[1] $X_{ijk} = \mu + \alpha_i + \delta_{k(i)} + \beta_j + (\alpha\beta)_{ij} + \lambda_{jk(i)}$

where μ, α_i, β_j, and $(\alpha\beta)_{ij}$ are fixed general effect, A effect, B effect, and A x B interaction effect parameters, respectively; $\delta_{k(i)}$ are random "subjects within A" effects, normally and independently distributed with $E[\delta_{k(i)}] = 0$ and variances, $\sigma^2_{S(A)}$; $\lambda_{jk(i)}$ are random "B by subjects within A" effects, normally and independently distributed with $E[\lambda_{jk(i)}] = 0$ and variance $\sigma^2_{BS(A)}$, with δ and λ independent.

Another model (see Collier (1958)), which is equivalent to expression [1], but more directly applicable to the present problem is:

[2] $X_{ijk} = \mu_{ij} + \phi_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \phi_{ijk}$

where: μ, α_i, β_j, $(\alpha\beta)_{ij}$ refer to the effects above in [1], but are more basically defined as functions of μ_{ij}, using the usual dot notation, i.e., $\mu = \bar{\mu}_{..}$, $\alpha_i = \bar{\mu}_{i.} - \mu_{..}$, $\beta_j = \bar{\mu}_{.j} - \mu_{..}$, $(\alpha\beta)_{ij} = \mu_{ij} - \bar{\mu}_{i.} - \bar{\mu}_{.j} + \bar{\mu}_{..}$ and ϕ_{ijk} are random residuals, IJK-variate normally distributed with $E[\phi_{ijk}] = 0$, $\mathrm{Var}(\phi_{ijk}) = \sigma^2$,

$\mathrm{Cov}(\phi_{ijk}, \phi_{i'j'k'}) = \rho\sigma^2$, for $i = i'$, $j \neq j'$, $k = k'$,

$= 0$, otherwise.

For the models in [1] and [2], respectively, variance and covariances are found to be:

$$[3] \quad \mathrm{Var}\,(X_{ijk}) = \sigma^2_{S(A)} + \sigma^2_{BS(A)} = \sigma^2$$
$$\mathrm{Cov}\,(X_{ijk}, X_{i'j'k'}) = \sigma^2_{S(A)} = \rho\sigma^2, \text{ for } i = i',\ j \neq j',\ k = k',$$
$$= 0, \text{ otherwise.}$$

The conventional analysis of variance, as indicated by Greenhouse and Geisser (1959) motivates the following statistics, with the indicated null distributions by which tests of null hypotheses on α_i, β_j and $(\alpha\beta)_{ij}$ can be made: $F_A = MSA/MS(S(A))$ is distributed as $F[I-1, I(K-1)]$; $F_B = MSB/MS(BS(A))$ is distributed as $F[J-1, I(J-1)(K-1]$; and $F_{AB} = MSAB/MS(BS(A))$ is distributed as $F[(I-1)(J-1), I(J-1)(K-1)]$.

The variances and covariances of ,[3] will be respecified to allow for possible heterogeneity. One specification which is reasonable is the following:

$$[4] \quad \mathrm{Var}\,(X_{ijk}) = \mathrm{Var}(\phi_{ijk}) = \sigma_{jj}$$
$$\mathrm{Cov}\,(X_{ijk}, X_{i'j'k'}) = \mathrm{Cov}(\phi_{ijk}, \phi_{i'j'k'}) = \sigma_{jj'} = \rho_{jj'}\sqrt{\sigma_{jj}\sigma_{j'j'}},\ i=i',\ j \neq j',\ k=k'$$
$$= 0, \text{ otherwise.}$$

Under these conditions the variance of any random observation, X_{ijk}, is constant for a given level of dimension A, but may possibly vary for different levels of B, whereas the covariance between a particular pair of random responses on dimension B for the same levels of i and k is constant over all subjects, but may possibly vary for different pairs of levels of B.

Strictly speaking, invoking the less stringent assumptions on variances and covariances of [4] renders inappropriate the previously given conventional F-tests. Although F_A still has the $F[I-1, I(K-1)]$ distribution, Box (1954) and Geisser and Greenhouse (1958) have shown that F_B is approximately distributed as $F[(J-1)\varepsilon,\ I(J-1)(K-1)\varepsilon]$ and F_{AB} has approximately, the $F[(I-1)(J-1)\varepsilon, I(J-1)(K-1)\varepsilon]$ distribution. The last two null distributions, then, are simply F-distributions with degrees of freedom reduced by a factor $\varepsilon = J^2(J^{-1}\sum_j \sigma_{jj} - J^{-2}\sigma_{..})^2/(J-1)$ $(\sum\sum_{jj'} \sigma^2_{jj'} - 2J^{-1}\sum_j \sigma^2_{j.} + \sigma^2_{..})$, wherein the dot notation is that normally employed. In demonstrating that the lower bound on ε is $(J-1)^{-1}$, (the upper bound is unity) Greenhouse and Geisser (1959) have provided conservative hypothesis tests on β_j

and $(\alpha\beta)_{ij}$. Thus, F_B is conservatively distributed as $F[1, I(K-1)]$ and F_{AB} is conservatively distributed as $F[(I-1), I(K-1)]$.

The major attempt here has been to obtain, by Monte Carlo sampling procedures, null distributions of F_A, F_B and F_{AB} and non-null distributions of F_B and F_{AB} under various specifications of σ_{jj} and $\sigma_{jj'}$, and to compare the conventional tests with Box's modified tests, and Greenhouse and Geisser's conservative tests. Comparisons will be made with respect to both size and power, although the power computations will be seen to be of a more selective nature and made realizing that the size values are not constant among procedures.

7.1.2 Procedures

The present results have been obtained by means of a computer program constructed to generate a random vector $X' = (X_1, X_2, \ldots, X_J)$ which has the J-variate normal distribution with arbitrarily specified mean vector, $U' = (U_1, U_2, \ldots, U_J)$, and covariance matrix, $C = [\sigma_{jj'}]$. This was accomplished by factoring C into LL', where L is a lower triangular matrix, and by using a subroutine which generated independent random vectors $Z' = (Z_1, Z_2, \ldots, Z_J)$ in which each Z is approximately independently and normally distributed with mean, zero, and variance, unity. From distribution theory it is known that the random vector, $X = U + LZ$ has the J-variate distribution with mean vector, U, and covariance matrix, C.

The general procedure employed in obtaining distributions of F_A, F_B and F_{AB} under various specifications of C was the following: 1) fix the value of C; 2) set the values of the mean vectors, U; 3) for each random sample of observations X_{ijk} compute F_A, F_B and F_{AB}; 4) repeat 2) and 3) until 1,000 samples have been generated and analyzed; 5) construct an empirical frequency distribution of these statistics using as class interval limits those values of each of three F-distributions--the conventional F-distribution, Box's modified F-distribution, and the conservative F-distribution specified by Greenhouse and Geisser--to obtain desired upper proportions, and 6) obtain an empirical distribution of an estimator of ε. In the case of the null distributions generated, the U were set to a constant thus yielding null values of α_i, β_j and $(\alpha\beta)_{ij}$. However, for power results, U were chosen to yield non-null values of β_j and $(\alpha\beta)_{ij}$ such that a power of .60 and .80 for a test size of .05 was obtained. Designation of U was made by means of Hartley and Pearson power charts with the knowledge that $\text{EMS}\left[BS(A)\right] = \sigma_w^2 = J^{-1}(\sum_j \sigma_{jj} - (J-1)^{-1} \sum_{j \neq j'} \sigma_{jj'})$ and $\phi^2 = IK\sum_j(\mu_{.j} - \bar{\mu}_{..})^2 / J\sigma_w^2$ for the main effect B and $\phi^2 = K\sum_i\sum_j(\mu_{ij} - \bar{\mu}_{i.} - \bar{\mu}_{.j} + \bar{\mu}_{..})^2 / \left[(I-1)(J-1)+1\right]\sigma_w^2$ for the A x B interaction. The computer program utilized is presented in Appendix C, p. 19.

Two specific cases have been considered, namely a configuration in which $I = 3$, $J = 4$, $K = 5$, and one in which $I = 3$, $J = 4$, $K = 15$. Thus, for each of a 15 subject and 45 subject configuration there are three levels of the "independent observation" (between subjects) dimension A, and four levels of the "dependent observation" (within subjects) dimension B. The numbers of subjects chosen above were judged by the authors to be acceptable typical sample sizes.

Selection of covariance matrices used in the study was made by first specifying variances, σ_{jj}, to be a) homogeneous and equal to 100; b) heterogeneous and equal to 100, 225, 400, and 900 and; c) heterogeneous and equal to 100, 400, 900 and 1600. Thereafter five different patterns of correlation coefficients, $\rho_{jj'}$ were chosen in conjunction with variance pattern a) in order to give a reasonable dispersion on ε. The resulting correlation matrix was repeated for variance sets b) and c). The covariance matrices produced by this procedure were the actual input matrices to the multivariate generator.

7.1.3 Results

Empirical results on the test size for the various cases are presented in Tables 19-21. Table 22 contains results on the empirical distribution of ε and Table 23 summarizes empirical power results. The variance and correlation structure (covariances are not given) for each of the cases considered is presented in Table 19. Thereafter, in Tables 20-23 the fifteen variance-correlation patterns are not repeated, but have been referred to as sets and indexed according to the specifications of Table 19.

7.1.3.1 Size of F_A-tests

The results in Table 19 serve as a gross check on the multivariate generator used in the study. They display, as would be expected, a general close agreement between empirical size values and comparable nominal α-values. The few divergences from theory can probably be accounted for by sampling disturbances or by the fact that the random variables generated were not quite normally distributed.

7.1.3.2 Size of F_B-tests

Table 20 provides information with which comparisons are made of F_B-tests. Underlying the tests are rejection rules defined on the F-distribution with a) no adjustment on degrees of freedom i.e., $\varepsilon = 1$; b) degrees of freedom reduced by a factor of true ε and c) degrees of freedom reduced by a factor of $(J-1)^{-1}$, the lower limit of ϵ. These three tests are referred to in Tables 20, 22 and 23 as the "unadjusted test", the "ε-adjusted test" and the "conservative test", respectively. Data in Table 20 indicate that, in general, the size of the unadjusted test is equal to or larger than the nominal α-value for both sample sizes, $K = 5$ and $K = 15$. Although the difference between the empirical size of the unadjusted test and nominal α increases somewhat systematically as the value of ε decreases, in general the discrepancy becomes important only for $\varepsilon \leq .64$. The arrangement of the results into five subsets, each with the same correlation structure for the three variance sets, seems to systematize the effect of decreasing ε fairly well, although not perfectly. Maximum differences between empirical test size and nominal α, for $\varepsilon \leq .64$ and $K = 5$ occur in set 35 and are .053, .055, .046 and .046 for α of .10, .05, .025, .01 respectively. In comparison these discrepancies are somewhat smaller for $K = 15$, namely, .046, .045, .042 and .027.

It is clear that, in almost every set, the ε-adjusted test achieves a correction on test size which brings empirical size to very close agreement with nominal α-values. Likewise, in the conservative test, the size of the test has been reduced to an extremely low value in comparison with nominal α. Indeed, for every set in Table 20, the empirical conservative test size is equal to or less than the nominal α.

7.1.3.3 Size of F_{AB}-tests

A pattern similar to that of F_B-tests is displayed in the F_{AB}-test size results of Table 21. Again for the unadjusted tests employing F_{AB}, the empirical size determinations are larger than nominal α-values. It appears that, for both 5 and 15 subjects per A level, the test sizes are, on the whole, slightly more discrepant from nominal α-values than those of F_B.

The ε-adjusted F_{AB}-tests and the conservative tests exhibit essentially the same size patterns in Table 21 as were observed in Table 20. Once more the ε-adjusted test size agrees closely with nominal α-values and the conservative test size appears to be extremely small as compared to nominal α-values.

7.1.3.4 The distribution of $\hat{\varepsilon}$

In view of the closeness with which ε-adjusted test size agrees with nominal α-values, it seemed reasonable to obtain information which might be useful in estimating ε. Consequently, in each of the 1000 null runs underlying the results of Tables 19 through 21, an empirical distribution was obtained on $\hat{\varepsilon}$, the sample estimator of ε. Since the runs were made under null α_i effects, an estimator of $\sigma_{jj'}$, say $\hat{\sigma}_{jj'}$, was defined as $\hat{\sigma}_{jj'} = \sum\sum_{IK}(X_{ijk}-\bar{X}_{.j.})(X_{ij'k}-\bar{X}_{.j.})/(IK-1)$. The estimator $\hat{\varepsilon}$ was obtained by substituting $\hat{\sigma}_{jj'}$ in the definition previously given for ε.

Distributions for $\hat{\varepsilon}$ were, in general, negatively skewed for high values of ε and positively skewed for low values of ε. Table 22 presents the empirical means and variances of the distributions of $\hat{\varepsilon}$. The indication is that $\hat{\varepsilon}$ becomes a more biased estimator as ε increases, such that for smaller values of ε (actually middle values in the possible range, .33 to 1,000) the bias for $\hat{\varepsilon}$ is almost null. Similarly for the smallest values of ε, the estimator $\hat{\varepsilon}$ has smallest variance. Since in practice the actual ε will probably not be much smaller than the lower values given in Table 22, it appears that there will be some possibility of precise estimation of a lower bound of ε. Thus, at this point, without considering the power of the test, it would be possible to utilize more accurately the ε-adjusted test by reducing the degrees of freedom through an estimator of ε. Results on power in the next section will be seen to throw some doubt on the advisability of this procedure.

7.1.3.5 Power of the F_B-tests and F_{AB}-tests

Power results for six variance-correlation patterns, each with five subjects per level of A, are given in Table 23. Non-null, equally spaced mean vectors were constructed in order to introduce, separately, non-null B and A x B effects and to yield a normal theory power of .60 and .80 for size .05. An example of the mean vectors actually used is included as a footnote to Table 23. Set groups 11, 21, 31 and 15, 25, 35 were chosen for power analysis, since they offer the widest range on ε among the several variance-correlation groups.

The information contained in Table 23 indicates, as would be expected from an inspection of empirical test size, the unadjusted tests to be uniformly more powerful than either the ε-adjusted or conservative tests. Only for large values of ε does the power of the ε-adjusted test approach that of the unadjusted test and, although the decrease in power for the ε-adjusted test is not as great as that of the conservative test, it is sizeable. For the unadjusted test, discrepancies between nominal and empirical power are seen to be greater for the F_B-test than for the F_{AB}-test. One explanation of this is that empirical size for the F_{AB}-test is generally greater than that of the F_B-test.

A somewhat interesting result is that the power of both the unadjusted and ε-adjusted tests decreases as ε decreases in spite of the fact that the size is increasing. One can possibly conclude that the non-null distributions of F_B and F_{AB}, for an aribtrary covariance matrix, are not approximated by either the unadjusted or ε-adjusted non-central F-distribution.

For nominal power of .60 the maximum decrease in power for unadjusted F_B-tests occurred in set 35, when ε was fixed at .45. In this case, the unadjusted test yielded a decrement of .092 from nominal power; the comparable decrement for the ε-adjusted test was .270 and the decrement for the conservative test was seen to be .297. Similarly, for nominal power of .80, one notes the maximum decrement for the F_B unadjusted test to be .147 with accompanying decrements for the ε-adjusted test and conservative test of .313 and .343, respectively. Comparable results hold for F_{AB} tests.

7.1.4 Conclusions

The results suggest that, except for extremely heterogeneous covariance matrices, the conventional F-test is optimal among the three procedures studied from the standpoint of both size and power. For populations characterized by relatively homogeneous covariances, as measured by the ε index, empirical test size obtained for the conventional, unadjusted test on the repeated measures effect and on the interaction effect was consistent with the nominal size of the conventional F-test assuming homogeneous covariance matrices. When the covariances were heterogeneous, the actual empirical test size was larger than the nominal size--noticeably so for extreme heterogeneity. The ε-adjusted test corrected very precisely for this bias when it existed and the conservative test over-corrected heavily, reducing empirical size to very small values when compared to nominal α.

The empirical analysis of power for the three test procedures, set nominally at .60 and .80 for $\alpha =$ .05, revealed that the conventional, unadjusted F-test gave uniformly higher power values than either the ε-adjusted or the conservative test. However, the power of the unadjusted test decreased as its size became increasingly larger than nominal α for increasing covariance heterogeneity. For covariance matrices which were not greatly heterogeneous the unadjusted, conventional test showed relatively good agrement with nominal values of power.

The ε-adjusted test, in reducing the size of the unadjusted test to almost nominal level, also reduced the power of this test to a coordinately lower value. As was to be expected from its extremely small size values, the conservative test was characterized by very small power values.

While keeping in mind the fact that the specific empirical results of this paper contain sampling error, one can draw fairly strong summaries from the data obtained. Among the three test procedures considered, the conventional, unadjusted F-test appears to have much to recommend it, as far as test size and power are concerned, except perhaps for extremely heterogeneous matrices. It is doubtful, in the bulk of psychological research, that covariance structures will be encountered with sufficient heterogeneity to cause a major disturbance of both size and power. Although the ε-adjusted test provides an excellent correction on test size, and employing an estimate of ε will be more adequately precise for small ε, the accompanying loss in power is not trivial.

7.2 Permutation analysis of the repeated measures design

7.2.1 Procedures

The randomization pattern defined for this design specified a two stage process in which a) K subjects are randomly assigned to each level of A and b) each subject is observed under J randomly assigned levels of B. The two stage permutation scheme appropriate to the IJK observations in this setup following from the randomization pattern is: a) the IK sets (of J observations each) associated with subjects are permuted into I groups of K sets in each group, and thereafter b) the J observations in each set are permuted within a subject.

For purposes of performing hypothesis tests connected with A, B and the AB interaction, reference is made to the EMS, under permutation, of Wilk and Kempthorne (1956) for the split-plot design. The suggestion there is that $F_A = MS(A)/MS(S(A))$, $F_B = MS(B)/MS(BS(A))$ and $F_{AB} = MS(AB)/MS(BS(A))$ are proper variance-ratios to consider for testing A, B, and AB, respectively. In these expressions MS(A) represents the usual analysis of variance mean square for effect A; MS(B) represents the similar mean square for effect B; MS(AB) refers to the mean square for the AB interaction; MS(S(A)) is the MS(between subjects within A) and MS(BS(A)) is the MS(B x subject interaction within A). The expectancy is that F_A, F_B and F_{AB} will have null permutation distributions which are approximately the $F[I-1, I(K-1)]$, $F[J-1, I(J-1)(K-1)]$ and $F[(I-1)(J-1), I(J-1)(K-1)]$ distributions, respectively.

As before, the desire is to investigate F-tests for basal response sets with a variety of attributes. Since the hypothesis test on A has already been studied in Section 4 under the completely randomized design, this portion of the analysis will be redundant. However, tests for B and AB under permutation have not to present knowledge been examined elsewhere. Therefore, the decision must be made concerning what characteristics of the basal responses should be varied in order to produce differential results on size and power of the F-tests. Motivated by the fact that, as regards factor A, we have an I x K completely randomized design and that, as regards factor B, we have a randomized block design with subjects as blocks the following conclusions were drawn.

With the previous realizations and with the knowledge that for the completely randomized design kurtosis has been shown to disturb size under permutation as compared to normal size (Pitman (1937b)), and that for the randomized block design intra-block variance heterogeneity has been shown to upset permutation test size (Pitman (1937a), and Welch (1937)), the decision was made to manipulate the kurtosis level of the total sample subject means and jointly to vary the intra-subject variance over subjects. Thus combinations of two levels of kurtosis, $\beta_2 = 3.0$ and 6.0 ($\beta_1 = .0$) and four levels of intra-subject variance heterogeneity Q = .96, 1.07, 1.29 and 1.40, were introduced as giving reasonable ranges on these coefficients for the dimensionality of the designs analyzed--I = 3, J = 4, K = 5. It should be noted that in this instance the definition of Q is that given previously with the number of intra-subject variances equal to 15. The dimensionality of this basal response set was judged sufficiently typical of experiments met in practice.

Both size and power of F_B and F_{AB} tests under permutation were obtained for the above eight combinations of β_2 and Q. The size results for F_A are reported incidentally. Following previous practice, non-null treatment effects were determined by means of Pearson and Hartley power charts in order to yield nominal power of .60 and .80 at nominal size of .05. Estimates of the error variance $\sigma^2_{BS(A)}$ were established as in previous sections. Permutation distributions were obtained on runs through the computer system previously described.

7.2.2 Results on size and power for the repeated measures design under permutation

Results on size for the specific repeated measures design considered appear in Table 24. The results for F_A are consistent with previous results for the completely randomized design. Leptokurtosis caused a drastic decrease in the size of the test under permutation as compared to its nominal α, since in this position of the analysis there are only five subjects in each of three A levels. Intra-subject variance, of course, had no effect on the A effect since subject means were being analyzed.

For the F_B-test, heterogeneity of subject variance, but not kurtosis, appears to have affected test size. There was good agreement between permutation and nominal test size for basal response data in which intra-subject variances were homogeneous, Q = .96, at both levels of kurtosis, although in every case test size under permutation was slightly above nominal value. Kurtosis, if it had any effect, seems to have caused a slight increase in empirical size above nominal values, at least for α of .10 and .05 The largest decreases in the size of F_B-tests were noted for cases in which Q = 1.29 and 1.40, i.e. response sets with rather heterogeneous variances. In those data sets, the empirical size under permutation for every case was smaller than the corresponding nominal values and an increase in Q was accompanied by a systematic and substantial decrease in size. For higher values of Q largest decreases in test size appear to have occurred at a nominal α of .05, .025 and .01. For example, at α of .05, test size under permutation was observed to be .021 and .020 for Q = 1.29 and 1.40, respectively. Only slight shifts in some of these test sizes occurred when β_2 was increased from 3.0 to 6.0.

F_{AB}-tests, for cases where basal responses exhibit variance homogeneity (Q = .96), seemed to display the same close agreement between test size under permutation and nominal size as F_B-tests. Again no systematic change was associated with the larger value of β_2. For basal responses with increasing heterogeneity of intra-subject variances, the empirical size of F_{AB} was, in general, lower than nominal values. Except for α = .10, F_{AB}-tests were affected to about the same degree as the F_B-tests. One notes no differential effect associated with β_2. However, at α = .10, Q = 1.29, β_2 = 3.0 there is a somewhat low value of the permutation test size.

Power results for F_B and F_{AB} reported in Table 24 indicate strongly that under permutation these tests agreed very well with their normal theory counter-values. One interesting result for F_{AB}-tests was that all empirical power values were larger than nominal values at $1-\beta = .80$. Otherwise no systematic trends were apparent in the power results.

7.2.3 Conclusions

One need only remark about results for F_B and F_{AB} since results on F_A are more fully treated in section 5. The conclusions can be stated simply that non-homogeneous intra-subject variance alone caused the size of the tests to become decreased as compared to nominal α and then only extremely large values of variance heterogeneity coefficient had any sizeable effect. These statements apply about equally for both F_B and F_{AB}. The power of F_B and F_{AB} tests under permutation is scarcely affected by varying K and/or β_2 and agrees closely with nominal power for the cases studied.

7.3 Permutation analysis of two alternative repeated measures designs.

7.3.1 Procedures

In this section the same configuration, a 3 x 4 x 5 layout, as that of section 7.2 was studied, however, the manner in which all treatment combinations were assigned to subjects was specified by means of two different designs. In the first instance the specifications of a split-plot design were followed. Thus there were assumed to be three levels of a whole-plot factor A, four levels of a split-plot factor B, but only five subjects (or levels of a replications dimension (R)). The randomization procedure consisted of first, randomly assigning the four levels of B within each level of A to that subject. This procedure was repeated anew for each subject separately.

According to Wilk and Kempthorne (1956), it is reasonable to expect that in such a split-plot design, F_A = MSA/MS(AxR), F_B = MSB/MS(BxR) and F_{AB} = MSAB/MS(AxBxR) will be approximately null-distributed under permutation as $F[3-1,(3-1)(5-1)]$, $F[4-1,(4-1)(5-1)]$ and $F[(3-1)(4-1),(3-1)(4-1)(5-1)]$, respectively. For the second design, the same format as for the split-plot was studied, but treatment combinations were assigned to a given subject (replication) as per the completely randomized design (or if one labeled replications as blocks, the randomized block design). Consequently a given subject received all 3 x 4 treatment combinations in a completely random order, this assignment being repeated anew for each subject in turn. Again according to Wilk and Kempthorne, the above defined F-ratios will be expected to be distributed as stated.

The size under permutation alone was obtained for the various tests for these two designs. In the case of the split-plot design (and motivated by results obtained for the randomized block design) three levels of intra whole-plot variance heterogeneity, $Q_A = .80$, 1.40 and 1.81 were defined in conjunction with three levels of intra split-plot variance heterogeneity, $Q_B = .96$, 1.29 and 1.40. Data were constructed to yield these nine combinations of Q_A and Q_B and were run for each of the two designs.

7.3.2 Results on test size for the split-plot repeated measures design

The permutation size of the F_A-test for this design given in Table 26 is fairly predictable if one considers the results of the randomized block design. Permutation size was systematically reduced as compared to nominal α almost solely as Q_A was increased to yield larger whole-plot variance heterogeneity. Similarly the F_B-test size under permutation was reduced systematically only as Q_B was increased. Even though the F_{AB} variance-ratio conceivably could be affected by both Q_A and Q_B, only Q_B had a reducing effect on the permutation size of the F_{AB}-test.

7.3.3 Results on test size for the completely randomized repeated measures design

The results on permutation size for the F_A-test are consistent with those of the randomized block design and indicate that for larger values of Q (see footnote in Table 27) and Q_A, the size is reduced appreciably as compared to nominal α.

F_A-test size was not affected by varying Q_B at $Q_A = .96$, but was reduced more heavily for larger values of Q corresponding to large values of both Q_A and Q_B. The permutation size of F_A and F_{AB} were similarly reduced with F_{AB} being somewhat more noticeably reduced.

7.3.4 Conclusions

For both designs it was clear that large variance heterogeneity within whole-plots, split-plots or replications lowers permutation test size. Once more the amount of heterogeneity required to cause a sizeable disturbance was large, particularly for moderate sample size.

8. Conclusions and implications

The computer program which has resulted from the present research project has proved to offer a mechanism for running permutation tests which is rapid and easy to use in a variety of experimental designs with equal frequency data configurations. Thus, if there is some question concerning the appropriateness of utilizing normal theory analysis of variance F-tests as approximations to exact permutation tests, this computer program can be employed. The output information from the program has been demonstrated in practice to be adequate for most experimental situations wherein a permutation test is applicable and the information obtained is that which is desired.

For circumstances where a large capacity computer is unavailable and where the running of usual F-tests is contemplated, for whatever reason, the research summarized in this report indicates, at least in a restricted sense, the inexactness which can be expected in using normal theory F-tests as approximations to permutation tests in a number of experimental designs. The overall conclusion appears to be that in most experimental circumstances the size under permutation of the analysis of variance F-tests usually employed agree very well with the nominal size of these F-tests under normal theory. This statement can be made even more strongly with respect to the power of the F-test under permutation. Almost always, extreme data matrices produced a reduction in size and power under permutation with the result that under these circumstances a conservative test will be expected.

Overall conclusions, as regards size and power of F-tests under permutation for specific designs, may be stated as follows:

1. In the case of the completely randomized design, skewness of the combined sample was found to have little effect on the size and/or power of F-tests under permutation. This was found to be true for almost all configuration sizes. However, highly leptokurtotic samples affected both size and power under permutation by reducing their values below nominal values. Increasing sample size, or more precisely the total possible number of permutations, actively diminished this effect such that in actual practice one would rarely find leptokurtosis sufficiently large to cause a noticeable disturbance. This remark can be made with considerable confidence for either single factor, or two factor completely randomized design with equal frequencies.

2. For the simple randomized block design with one observation per cell, one concludes through the present research that heterogeneity of block variances is accompanied by a systematic reduction in size (and slight reduction of power) for the F_t under permutation as compared to nominal size and power values. However, only when the configuration is small does variance heterogeneity become a major disturbing element. In addition, non-null treatment x block interaction further reduces both size and power coordinately.

For the randomized block design with more than one observation per block-treatment cell, the permutation size of the F-test on treatments using MS(within cells) as denominator is affected only slightly by variations in block variance heterogeneity and/or within block leptokurtosis. The conclusion is that only when both variance heterogeneity and leptokurtosis are large will the agreement between permutation and nominal size suffer. Using MS(treatment x block) as error, however, one experiences a reduction in permutation size as block variance heterogeneity is increased. Thus, there seems to be little justification for not using the within cells error term for such a situation. Similar remarks hold for tests on treatment x block interaction.

Again power is much less affected by increases in variance heterogeneity or kurtosis. For both treatments (with MS(within cells) as error) and treatment x block interaction tests, power under permutation in comparison with nominal values is reduced only for very large values of both variance heterogeneity and positive kurtosis, although the power values corresponding to $\alpha = .01$ were generally reduced more than at .05. Employing MS(treatment x block) as error for the treatments F-test results in a decrease in power under permutation, more so if, in fact, there is a non-null treatment x block interaction.

Overall one would be advised to apply the randomized block design with more than one observation per cell if at all possible because of the possible deleterious effects of unknown treatment x block interaction. Certainly, F-tests in randomized block designs with multiple observations per cell exhibit strong agreement between size and power under permutation and under normal theory.

3. The investigations of repeated measures designs studied by means of sampling from approximate multivariate populations with various covariance matrices, indicate that unless the variance-covariance matrix is exceedingly non-patterned the usual F-tests on both the repeated factor and the interaction between non-repeated and repeated factors is reasonably exact, as regards size and power. For cases in which high heterogeneity exists in the covariance matrix, large increase over nominal values in size and a decrease in power of the usual F-test is to be expected. Whereas a test based on modified F-distributions can be expected to reduce size, a concomitant and quite large reduction in power will also be experienced.

4. Repeated measures designs, in the case where each subject is randomly assigned all levels of one factor at a given level of a second factor, can be expected to experience a reduction in the permutation size for the above two tests as compared to nominal values. This is particularly true if there exists large subject variance heterogeneity. Power of these tests under permutation is affected very little by larger variance heterogeneity levels.

However, for the case with the same total number of observations as in the latter repeated measures design and where each subject is randomly administered _all_ treatment combinations of two factors in a split-plot or a completely randomized design, the size of the tests under permutation is reduced as compared to nominal values. This reduction is particularly high if variance heterogeneity for either of the treatment factors is large.

5. It is to be observed, in general, that for most experimental cases encountered in actual practice, the normal theory F-tests will offer excellent approximations to permutation tests. Thus, it will be expected that the size and power of F-tests under permutation probability bases will, except for extreme configurations, agree extremely well with normal theory values.

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APPENDIX A, TABLES *

	Expected proportion					
	.80	.60	.10	.05	.025	.01
* Standard error of obtained proportions	.0126	.0155	.0095	.0069	.0049	.0031

Table 1

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_1 = 0]$,
for two replications of the 2 x 6 and 2 x 12 completely randomized design
under various combinations of skewness and kurtosis .

			Kurtosis (β_2)											
			2.0				3.0				4.0			
Skewness(β_1)	Design	Rep	α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
0.0	2 x 6	1	.089	.049	.032	.013	.100	.048	.023	.003	.102	.033	.010	.000
		2	.098	.056	.031	.018	.084	.035	.019	.002	.112	.034	.011	.000
	2 x 12	1	.102	.050	.022	.009	.101	.056	.028	.013	.106	.055	.028	.013
		2	.090	.051	.027	.009	.109	.060	.029	.008	.086	.038	.020	.007
0.5	2 x 6	1	.112	.071	.036	.009	.083	.043	.023	.008	.112	.058	.010	.000
		2	.111	.063	.030	.023	.100	.041	.017	.007	.102	.028	.012	.002
	2 x 12	1	.114	.063	.028	.011	.123	.055	.026	.008	.107	.046	.020	.010
		2	.106	.045	.027	.012	.106	.062	.035	.010	.111	.060	.031	.014
1.0	2 x 6	1	.072	.000	.000	.000 **	.089	.036	.015	.000	.075	.041	.013	.005
		2	.089	.011	.000	.000	.108	.058	.021	.012	.069	.032	.011	.000
	2 x 12	1	.122	.058	.029	.007	.104	.056	.026	.010	.092	.034	.020	.009
		2	.105	.053	.024	.005	.100	.050	.023	.006	.100	.053	.022	.006

**NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 2

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1, n_2) | T_i = 0]$, for two replications of the 3 x 6 and 3 x 12 completely randomized design under various combinations of skewness and kurtosis.

			Kurtosis (β_2)											
			2.0				3.0				4.0			
Skewness(β_1)	Design	Rep	α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
0.0	3 x 6	1	.093	.052	.030	.016	.091	.042	.017	.007	.097	.048	.021	.007
		2	.107	.052	.022	.011	.078	.038	.018	.006	.107	.052	.019	.007
	3 x 12	1	.096	.050	.026	.008	.093	.044	.022	.008	.113	.055	.017	.007
		2	.100	.056	.025	.014	.088	.048	.023	.005	.095	.047	.020	.005
0.5	3 x 6	1	.119	.061	.034	.013	.097	.042	.019	.010	.089	.040	.016	.007
		2	.111	.052	.025	.010	.098	.048	.023	.007	.094	.048	.020	.005
	3 x 12	1	.102	.047	.020	.010	.076	.037	.019	.007	.106	.051	.024	.008
		2	.104	.058	.030	.006	.091	.043	.019	.008	.092	.045	.023	.009
1.0	3 x 6	1	.107	.034	.008	.000 **	.098	.049	.023	.015	.107	.041	.016	.004
		2	.077	.037	.013	.003	.083	.037	.016	.006	.091	.029	.013	.002
	3 x 12	1	.108	.053	.027	.006	.108	.040	.024	.009	.097	.049	.026	.006
		2	.099	.051	.023	.006	.090	.051	.024	.008	.091	.042	.019	.005

** NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space, the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 3 Size of permutation test, $P[F_t > F_{1-\alpha}(n_1, n_2) | T_i = 0]$, for effects A, B and AB in a 2 x 2 x 6 completelv randomized design under various combinations of skewness and kurtosis.

			Kurtosis (β_2)											
			2.0				3.0				4.0			
Skewness(β_1)	Effect	Rep	α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
0.0	A	1	.094	.053	.027	.015	.121	.058	.026	.007	.096	.049	.023	.010
		2	.107	.058	.034	.017	.103	.056	.031	.013	.097	.041	.022	.009
	B	1	.097	.055	.026	.009	.116	.063	.033	.016	.097	.054	.021	.008
		2	.114	.056	.029	.012	.106	.051	.022	.011	.095	.054	.025	.009
	AB	1	.102	.048	.027	.012	.123	.057	.025	.010	.126	.046	.021	.008
		2	.109	.063	.033	.014	.087	.046	.022	.011	.096	.044	.017	.006
0.5	A	1	.112	.058	.026	.014	.090	.054	.022	.010	.106	.048	.020	.006
		2	.096	.054	.029	.011	.123	.060	.031	.011	.104	.045	.025	.007
	B	1	.097	.050	.024	.010	.095	.047	.027	.014	.104	.053	.028	.010
		2	.072	.037	.021	.006	.090	.043	.025	.012	.096	.049	.031	.010
	AB	1	.093	.047	.027	.012	.102	.057	.029	.014	.092	.047	.023	.010
		2	.104	.052	.026	.011	.094	.047	.025	.008	.107	.054	.025	.007
1.0	A	1	.116	.054	.025	** .004	.093	.042	.020	.009	.086	.039	.018	.006
		2	.086	.034	.011	.004	.095	.052	.027	.010	.116	.045	.023	.008
	B	1	.101	.051	.016	.006	.103	.045	.021	.013	.096	.042	.022	.008
		2	.101	.047	.017	.001	.076	.039	.018	.006	.091	.042	.018	.005
	AB	1	.107	.045	.019	.006	.086	.040	.017	.007	.107	.047	.020	.008
		2	.107	.045	.012	.006	.095	.060	.026	.008	.094	.048	.022	.007

** NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space, the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 4

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = 0]$,
for effects A, B and AB in a 2 x 2 x 12 completely randomized design
under various combinations of skewness and kurtosis.

			Kurtosis (β_2)											
			2.0				3.0				4.0			
Skewness (β_1)	Effect	Rep	α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
0.0	A	1	.105	.061	.031	.018	.098	.045	.019	.011	.105	.057	.027	.009
		2	.103	.040	.024	.011	.092	.045	.029	.015	.103	.058	.032	.010
	B	1	.100	.047	.028	.011	.088	.045	.026	.008	.090	.044	.017	.009
		2	.101	.051	.026	.014	.099	.052	.024	.009	.108	.061	.030	.009
	AB	1	.092	.048	.020	.007	.096	.048	.024	.008	.105	.060	.025	.011
		2	.100	.047	.023	.008	.113	.065	.034	.015	.080	.042	.020	.010
0.5	A	1	.098	.049	.024	.011	.096	.048	.027	.010	.102	.044	.024	.007
		2	.100	.049	.026	.008	.098	.046	.028	.015	.093	.050	.027	.014
	B	1	.098	.047	.024	.013	.098	.048	.024	.013	.101	.052	.027	.011
		2	.099	.057	.025	.014	.093	.048	.022	.011	.104	.049	.023	.008
	AB	1	.098	.054	.027	.009	.106	.049	.031	.011	.098	.045	.019	.008
		2	.088	.040	.021	.007	.083	.050	.026	.011	.095	.042	.022	.006
1.0	A	1	.105	.050	.020	.008 **	.089	.040	.017	.004	.100	.051	.021	.006
		2	.115	.057	.028	.009	.093	.049	.020	.007	.094	.041	.018	.007
	B	1	.100	.041	.022	.007	.093	.048	.028	.011	.094	.042	.018	.005
		2	.094	.047	.024	.011	.089	.048	.022	.008	.092	.051	.026	.013
	AB	1	.121	.064	.029	.008	.115	.054	.025	.009	.106	.055	.026	.012
		2	.112	.056	.030	.010	.111	.045	.026	.011	.091	.047	.019	.006

** NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ heve been presented in this cell.

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Table 5

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) \mid T_i = 0]$, for effects A, B and AB in a 3 x 2 x 6 completely randomized design under various combinations of skewness and kurtosis.

			Kurtosis (β_2)											
			2.0				3.0				4.0			
Skewness(β_1)	Effect	Rep	α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
0.0	A	1	.094	.048	.021	.010	.091	.050	.028	.012	.097	.042	.023	.009
		2	.101	.051	.023	.014	.100	.054	.023	.011	.106	.059	.022	.004
	B	1	.095	.046	.025	.013	.088	.039	.019	.006	.104	.050	.022	.008
		2	.114	.049	.031	.017	.113	.056	.030	.015	.110	.053	.026	.013
	AB	1	.090	.040	.021	.009	.095	.051	.017	.006	.103	.061	.027	.008
		2	.109	.049	.017	.005	.091	.047	.022	.011	.091	.044	.020	.008
0.5	A	1	.107	.062	.029	.016	.106	.052	.032	.014	.103	.047	.021	.008
		2	.108	.053	.032	.014	.095	.044	.025	.014	.097	.056	.033	.013
	B	1	.100	.054	.031	.013	.111	.056	.029	.013	.097	.047	.026	.013
		2	.093	.050	.023	.008	.120	.052	.025	.011	.093	.048	.030	.007
	AB	1	.104	.062	.036	.018	.098	.049	.027	.009	.083	.047	.023	.013
		2	.110	.056	.032	.013	.100	.052	.029	.015	.097	.051	.021	.006
1.0	A	1	.103	.042	.020	.005 **	.110	.058	.032	.013	.096	.044	.022	.008
		2	.114	.056	.025	.009	.096	.047	.031	.014	.099	.047	.017	.003
	B	1	.098	.045	.017	.004	.097	.052	.032	.018	.099	.048	.028	.010
		2	.105	.056	.018	.006	.076	.035	.019	.009	.095	.049	.022	.007
	AB	1	.092	.039	.017	.006	.108	.066	.040	.020	.090	.040	.021	.009
		2	.092	.038	.018	.006	.095	.046	.025	.009	.099	.043	.016	.005

** **NOTE** - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space, the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 6

Size of permutation test, $P[F_t \geq F_{\cdot}(n_1, n_2) \mid T_i = 0]$, for effects A, B and AB in a 3 x 2 x 2 completely randomized design under various combinations of skewness and kurtosis.

			Kurtosis (β_2)											
			2.0				3.0				4.0			
Skewness(β_1)	Effect	Rep	α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
0.0	A	1	.099	.051	.017	.006	.106	.050	.024	.008	.085	.036	.011	.004
		2	.073	.039	.019	.006	.100	.047	.029	.009	.105	.054	.029	.009
	B	1	.074	.048	.021	.010	.105	.052	.025	.005	.103	.057	.024	.008
		2	.108	.055	.032	.013	.083	.040	.017	.011	.101	.051	.022	.010
	AB	1	.096	.048	.026	.012	.093	.047	.023	.007	.101	.055	.031	.012
		2	.114	.058	.031	.010	.092	.048	.028	.008	.124	.068	.031	.013
0.5	A	1	.111	.056	.026	.013	.092	.053	.020	.007	.092	.048	.021	.010
		2	.107	.055	.025	.013	.115	.058	.029	.011	.086	.046	.015	.005
	B	1	.083	.051	.025	.008	.088	.047	.023	.010	.111	.059	.031	.015
		2	.087	.049	.026	.008	.093	.048	.032	.010	.108	.051	.025	.009
	AB	1	.100	.051	.028	.010	.097	.043	.022	.008	.100	.040	.023	.011
		2	.099	.053	.030	.007	.097	.038	.016	.004	.091	.045	.029	.016
1.0	A	1	.104	.035	.018	.004**	.112	.059	.024	.011	.106	.055	.027	.016
		2	.095	.048	.019	.003	.110	.057	.028	.016	.077	.039	.018	.004
	B	1	.113	.053	.028	.011	.114	.062	.040	.015	.101	.049	.026	.015
		2	.085	.047	.022	.008	.095	.047	.020	.008	.084	.040	.021	.009
	AB	1	.091	.039	.021	.008	.099	.047	.019	.006	.089	.052	.022	.009
		2	.120	.066	.029	.012	.088	.051	.023	.012	.100	.043	.026	.009

**NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space, the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 7

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = 0]$, for two replications of the 2 x 12, 3 x 12, 2 x 2 x 6, and 3 x 2 x 6 completely randomized design for $\beta_1 = 0.0$ and $\beta_2 = 10.0$

Design	Effect	Rep	α			
			.10	.05	.025	.01
2 x 12		1	.108	.000	.000	.000
		2	.106	.000	.000	.000
3 x 12		1	.076	.027	.005	.002
		2	.086	.024	.006	.002
2 x 2 x 6	A	1	.124	.000	.000	.000
		2	.118	.000	.000	.000
	B	1	.133	.000	.000	.000
		2	.115	.000	.000	.000
	AB	1	.125	.000	.000	.000
		2	.118	.000	.000	.000
3 x 2 x 6	A	1	.083	.025	.007	.001
		2	.091	.029	.010	.001
	B	1	.107	.039	.009	.001
		2	.106	.039	.014	.001
	AB	1	.089	.023	.004	.000
		2	.087	.034	.014	.002

Table 8 Power of the F-test under permutation, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i \neq 0]$, for the 2 x 6 and 2 x 12 completely randomized design under various combinations of skewness and kurtosis. Results relate to rep 1, null run data for nominal power set at .60 and .80 for α = .05.

		Nominal Power		Kurtosis (β_2)					
				2.0		3.0		4.0	
Skewness(β_1)	Design	α .05	.01	.05	.01	.05	.01	.05	.01
0.0	2 x 6	.60	.32	.549*	.239	.534	.234	.556	.227
		.80	.51	.798	.436	.794	.438	.785	.417
	2 x 12	.60	.35	.546	.275	.603	.287	.547	.277
		.80	.54	.766	.493	.802	.552	.786	.494
0.5	2 x 6	.60	.32	.493	.205	.516	.230	.528	.255
		.80	.51	.801	.392	.775	.406	.799	.446
	2 x 12	.60	.35	.512	.243	.574	.284	.571	.297
		.80	.54	.733	.421	.785	.523	.785	.518
1.0	2 x 6	.60	.32	.429	.164**	.581	.234	.509	.232
		.80	.51	.758	.257	.819	.429	.750	.440
	2 x 12	.60	.45	.500	.205	.553	.307	.578	.317
		.80	.54	.743	.404	.786	.502	.813	.540

**NOTE - Basal responses for β_1 = 1.0, β_2 = 2.0 were unobtainable. In order to conserve space, the results for basal responses for β_1 = 0.0, β_2 = 5.0 have been presented in this cell.

*Data and non-null treatment effects used for the 2 x 6 design, β_1 = 0.0, β_2 = 2.0

Data

218	186
182	193
195	206
197	194
213	178
207	202

	Power = .60		Power = .80	
	ϕ^2	Treatment Effects	ϕ	Treatment Effects
A_1	2.99	11.37	4.84	9.03
A_2		28.63		30.97

Table 9

Power of the F-test under permutation, $P[F_t > F_{1-\alpha}(n_1, n_2) | T_i \neq 0]$, for the 3 x 6 and 3 x 12 completely randomized design under various combinations of skewness and kurtosis. Results relate to rep 1, null run data for nominal power set at .60 and .80 for $\alpha = .05$.

		Nominal Power		Kurtosis (β_2)					
				2.0		3.0		4.0	
Skewness (β_1)	Design	α.05	.01	.05	.01	.05	.01	.05	.01
0.0	3 x 6	.60	.32	.547	.246	.526	.243	.552	.263
		.80	.53	.765	.441	.773	.427	.792	.455
	3 x 12	.60	.32	.600	.291	.550	.304	.605	.325
		.80	.55	.788	.556	.779	.512	.789	.547
0.5	3 x 6	.60	.32	.567	.281	.549	.264	.554	.254
		.80	.53	.787	.458	.775	.457	.793	.444
	3 x 12	.60	.32	.567	.304	.566	.304	.583	.301
		.80	.55	.757	.496	.793	.517	.787	.529
1.0	3 x 6	.60	.32	.507	.209 **	.538	.260	.576	.276
		.80	.53	.741	.354	.785	.438	.792	.469
	3 x 12	.60	.32	.541	.296	.548	.270	.593	.330
		.80	.55	.768	.480	.775	.487	.807	.557

**NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space, the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 10

Power of the F-test under permutation, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_1 \neq 0]$, for the 2 x 12, 3 x 12, 2 x 2 x 6 and 3 x 2 x 6 completely randomized design for $\beta_2 = 10.0$ and $\beta_1 = 0.0$. Results relate to rep 1, null run data, for nominal power set at .60 and .80 for $\alpha = .05$.

Design	Effect	Nominal Power			
		α .05	.01	.05	.01
2 x 12		.60	.35	.577	.246
		.80	.54	.773	.396
3 x 12		.60	.32	.586	.315
		.80	.55	.805	.515
2 x 2 x 6	A	.60	.35	.610	.302
		.80	.55	.748	.510
	AB	.60	.35	.624	.285
		.80	.55	.743	.491
3 x 2 x 6	A	.60	.34	.593	.326
		.80	.56	.786	.542
	AB	.60	.34	.589	.319
		.80	.56	.779	.521

Table 11 Power of the F-test under permutation, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i \neq 0]$, for the 2 x 2 x 6 completely randomized design under various combinations of skewness and kurtosis. Results relate to rep 1, null run data for nominal power set at .60 and .80 for α = .05.

		Nominal Power		Kurtosis(β_2)					
				2.0		3.0		4.0	
Skewness(β_1)	Effect	α .05	.01	.05	.01	.05	.01	.05	.01
0.0	A	.60	.35	.582*	.295	.577	.293	.571	.289
		.80	.55	.788	.492	.778	.520	.787	.504
	AB	.60	.35	.593	.301	.563	.275	.565	.288
		.80	.55	.802	.508	.819	.526	.796	.521
0.5	A	.60	.35	.586	.305	.588	.295	.574	.313
		.80	.55	.797	.514	.796	.507	.805	.534
	AB	.60	.35	.573	.298	.592	.311	.589	.298
		.80	.55	.801	.525	.800	.507	.796	.522
1.0	A	.60	.35	.593	.307**	.573	.288	.584	.286
		.80	.55	.799	.512	.781	.501	.792	.515
	AB	.60	.35	.586	.306	.595	.322	.581	.313
		.80	.55	.792	.521	.783	.519	.810	.531

**NOTE - Basal responses for β_1 = 1.0, β_2 = 2.0 were unobtainable. In order to conserve space, the results for basal responses for β_1 = 0.0, β_2 = 5.0 have been presented in this cell.

*Data and non-null treatment effects used for the 2 x 2 x 6 design with β_1 = 0.0, β_2 = 2.0

Data				
	204	206	212	193
	196	211	202	200
	189	210	180	213
	208	204	194	199
	186	190	183	192
	204	205	193	196

Non-null Effect	Level of B	Power = .60			Power = .80		
		ϕ^2	A_1	A_2	ϕ^2	A_1	A_2
A	Each B_i	2.69	15.58	24.42	4.33	14.39	25.61
AB	B_1	2.69	15.58	24.42	4.33	14.39	25.61
	B_2		24.42	15.58		25.61	14.39

Table 12

Power of the F-test under permutation, $P[F_t > F_{1-\alpha}(n_1, n_2) \mid T_i \neq 0]$,
for the 3 x 2 x 6 completely randomized design under various combinations of skewness and kurtosis.
Results relate to rep 1, null run data, for nominal power set at .60 and .80 for α = .05.

		Nominal Power		Kurtosis(β_2)					
				2.0		3.0		4.0	
Skewness(β_1)	Effect	α .05	.01	.05	.01	.05	.01	.05	.01
0.0	A	.60 .80	.34 .56	.610 .797	.316 .545	.618 .798	.337 .552	.605 .797	.316 .540
	AB	.60 .80	.34 .56	.563 .760	.278 .525	.607 .819	.311 .561	.575 .786	.307 .525
0.5	A	.60 .80	.34 .56	.600 .800	.317 .550	.606 .791	.356 .526	.586 .806	.313 .560
	AB	.60 .80	.34 .56	.582 .791	.298 .553	.604 .802	.305 .555	.593 .770	.305 .507
1.0	A	.60 .80	.34 .56	.587 .813	** .305 .528	.571 .783	.277 .550	.570 .770	.301 .531
	AB	.60 .80	.34 .56	.592 .810	.315 .548	.599 .769	.345 .537	.602 .784	.322 .522

**NOTE - Basal responses for $\beta_1 = 1.0$, $\beta_2 = 2.0$ were unobtainable. In order to conserve space, the results for basal responses for $\beta_1 = 0.0$, $\beta_2 = 5.0$ have been presented in this cell.

Table 13

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2)\ T_i = 0]$,
for the 3 x 5 and 3 x 12 randomized block design with one observation per cell.
Results are reported on basal responses with three intra-block variance patterns
for null and non-null treatment x block interaction.

				Treatment x block interaction null				Treatment x block interaction non-null											
								1				2				3			
Variance*	Design	Q	Rep	α .10	.05	.025	.01	α .10 **	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
I	3 x 5	.80	1	.106	.063	.040	.022	.088	.048	.028	.017	.057	.022	.013	.003	.017	.003	.003	.001
			2	.117	.071	.045	.033												
	3 x 12	.92	1	.104	.061	.031	.014	.078	.036	.021	.008	.038	.019	.009	.001	.018	.002	.001	.000
			2	.105	.058	.035	.016												
II	3 x 5	1.41	1	.070	.018	.009	.000	.074	.037	.024	.007	.069	.040	.024	.015	.043	.024	.014	.005
			2	.069	.012	.000	.000												
	3 x 12	1.29	1	.100	.043	.010	.001	.074	.036	.019	.008	.047	.027	.017	.005	.025	.014	.005	.002
			2	.097	.035	.004	.001												
III	3 x 5	1.81	1	.061	.004	.000	.000	.054	.035	.029	.015	.057	.034	.029	.018	.036	.023	.014	.003
			2	.022	.000	.000	.000												
	3 x 12	1.43	1	.091	.023	.003	.000	.073	.040	.018	.004	.051	.031	.014	.003	.030	.010	.003	.001
			2	.078	.028	.007	.001												

* For specification of intra-block variance patterns see Table 13a.

** For examples of non-null treatment x block interaction specifications see Table 13a.

Table 13a

Specification of intra-block variance patterns

The variances are, within a constant to adjust for average variance, as follows:

Design	Var	1	2	3	4	5	6	7	8	9	10	11	12
3 x 5	I	1.0	1.0	1.0	1.0	1.0							
	II	1.0	3.0	9.0	27.0	81.0							
	III	1.0	4.0	16.0	64.0	256.0							
3 x 12	I	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	II	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0
	III	1.0	2.3	5.3	12.2	28.0	64.4	148.0	340.5	783.1	1801.2	4141.7	9528.1

Non-null treatment x block interaction
Randomized block design

Design	Spec 1	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}
3 x 5	T_1	3.0	6.0	6.0	12.0	15.0							
	T_2	9.0	9.0	9.0	9.0	9.0							
	T_3	15.0	12.0	9.0	6.0	3.0							
3 x 12	T_1	5.9	11.8	17.7	23.6	29.6	35.5	41.4	47.3	53.2	59.1	65.0	70.9
	T_2	38.4	38.4	38.4	38.4	38.4	38.4	38.4	38.4	38.4	38.4	38.4	38.4
	T_3	70.9	65.0	59.1	53.2	47.3	41.4	35.5	29.6	23.6	17.7	11.8	5.9

Table 14

Size of the permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = C]$, on A and AB for the 3 x 3 randomized block design with 6 observations per cell. Results are reported on basal responses for various combinations of intra-block variance patterns and values of kurtosis.

					Kurtosis (β_2)											
					3.0				5.0				7.0			
Variance *	Q	Effect	Error	α	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
I	.67	A	AB		.105	.053	.026	.016	.109	.060	.031	.016	.100	.050	.029	.015
		A	S(AB)		.092	.040	.020	.008	.104	.052	.024	.007	.102	.043	.016	.006
		AB	S(AB)		.093	.037	.016	.009	.099	.046	.014	.003	.076	.028	.011	.000
II	1.12	A	AB		.025	.001	.000	.000	.035	.004	.001	.000	.028	.001	.000	.000
		A	S(AB)		.103	.051	.026	.012	.108	.055	.032	.011	.115	.053	.017	.002
		AB	S(AB)		.120	.067	.042	.021	.109	.055	.027	.015	.075	.031	.009	.003
III	3.37	A	AB		.012	.003	.000	.000	.013	.004	.001	.000	.021	.007	.001	.000
		A	S(AB)		.097	.060	.036	.018	.120	.058	.037	.021	.088	.028	.003	.000
		AB	S(AB)		.137	.081	.058	.037	.138	.087	.056	.035	.118	.045	.013	.001

* Specification of variance patterns used:

Variance Specification	Block Variances 1	2	3
I	37	37	37
II	1	38	75
III	1	10	100

Table 15

Power of the F-test, $P\left[F_t > F_{1-\alpha}(n_1,n_2) \mid T_i \neq 0\right]$, for the 3 x 5 and 3 x 12 randomized block design with one observation per cell. Results are reported on basal responses with three intra-block variance patterns for null and non-null treatment x block interactions. Nominal power set at .60 and .80 for $\alpha = .05$.

			Nominal Power		Treatment x block interaction null		Treatment x block interaction non-null					
							1		2		3	
Variance	Design	Q	α .05	.01	.05	.01	.05	.01	.05	.01	.05	.01
I	3 x 5	.80	.60	.28	.535 *	.215	.470	.172	.357	.111	.211	.059
			.80	.48	.809	.371	.706	.317	.567	.211	.404	.118
	3 x 12	.92	.60	.32	.560	.278	.463	.203	.316	.106	.185	.038
			.80	.54	.771	.496	.683	.390	.547	.235	.351	.107
II	3 x 5	1.41	.60	.28	.557	.205	.440	.050	.213	.002	.040	.000
			.80	.48	.784	.407	.894	.193	.676	.067	.301	.008
	3 x 12	1.29	.60	.32	.569	.299	.468	.106	.256	.006	.038	.000
			.80	.54	.774	.506	.780	.340	.593	.081	.269	.002
III	3 x 5	1.81	.60	.28	.574	.197	.427	.023	.149	.000	.003	.000
			.80	.48	.793	.423	.916	.151	.676	.034	.210	.001
	3 x 12	1.43	.60	.32	.584	.326	.451	.090	.211	.002	.024	.000
			.80	.54	.802	.520	.788	.329	.584	.059	.214	.000

* Non-null treatment effects used for the 3 x 5 randomized block design with "variance I" specification

Power	ϕ^2	T_1	T_2	T_3
.60	3.03	9.79	0.00	-9.79
.80	4.80	12.32	0.00	-16.32

Table 16

Power of the F-test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = 0]$, for the 3 x 5 and 3 x 12 randomized block design with one observation per cell. Results are reported on basal responses with three intra-block variance patterns for null and non-null treatment x block interactions with the interactions taken into account in the calculations of treatment effects. Nominal power set at .60 and .80 for α = .05.

			Nominal Power		Treatment x block interaction null		Treatment x block interaction non-null					
							1		2		3	
Variance	Design	Q	α .05	.01	.05	.01	.05	.01	.05	.01	.05	.01
I	3 x 5	.80	.60	.28	.535	.215	.543	.208	.555	.208	.556	.180
			.80	.48	.809	.371	.805	.409	.811	.416	.839	.400
	3 x 12	.92	.60	.32	.560	.278	.562	.275	.568	.274	.568	.274
			.80	.54	.771	.496	.770	.498	.771	.502	.777	.500
II	3 x 5	1.41	.60	.28	.557	.205	.614	.084	.683	.073	.703	.056
			.80	.48	.784	.407	.991	.366	.998	.380	.999	.372
	3 x 12	1.29	.60	.32	.569	.299	.556	.274	.561	.264	.570	.250
			.80	.54	.774	.506	.804	.500	.829	.495	.847	.500
III	3 x 5	1.81	.60	.28	.574	.197	.645	.050	.728	.042	.747	.044
			.80	.48	.793	.423	1.000	.370	1.000	.374	1.000	.364
	3 x 12	1.43	.60	.32	.584	.326	.590	.306	.599	.292	.608	.269
			.80	.54	.802	.520	.822	.526	.842	.528	.870	.533

Table 17

Power of the F-test under permutation, $P\left[F_t > F_{1-\alpha}(n_1 . n_2) \mid T_i \neq 0\right]$, on A and AB for the 3 x 3 randomized block design with 6 observations per cell. Results are reported on basal responses for various combinations of intra-block variance patterns and values of kurtosis. Nominal power set at .60 for $\alpha = .05$.

				Nominal Power		Kurtosis (β_2)					
						3.0		5.0		7.0	
Variance	Q	Effect	Error	α .05	.01	.05	.01	.05	.01	.05	.01
I	.67	A	AB	.60	.15	.521**	.179	.529	.184	.509	.183
		A	AB*	.60	.15	.503	.100	.511	.103	.521	.092
		A	S(AB)	.60	.34	.603	.349	.527	.335	.595	.334
		AB	S(AB)	.60	.34	.603	.353	.591	.324	.594	.338
II	1.12	A	AB	.60	.15	.571	.221	.558	.200	.567	.197
		A	AB*	.60	.15	.502	.132	.507	.144	.536	.132
		A	S(AB)	.60	.34	.594	.342	.574	.332	.586	.341
		AB	S(AB)	.60	.34	.583	.330	.573	.314	.558	.325
III	3.37	A	AB	.60	.15	.596	.251	.620	.257	.579	.279
		A	AB*	.60	.15	.522	.189	.543	.198	.566	.158
		A	S(AB)	.60	.34	.582	.329	.568	.330	.588	.343
		AB	S(AB)	.60	.34	.545	.331	.519	.314	.508	.331

* AB interaction was non-null for this run.

** Non-null treatment effects used for the randomized block design with six observations per cell for data with kurtosis = 3.0 and "variance I" specification.

Error term	Non-null Effect	Level of B	ϕ^2	A_1	A_2	A_3
AB	A	Each B_i	4.20	-3.66	0.00	3.66
S(AB)	A	Each B_i	2.25	-2.71	0.00	2.71
S(AB)	AB	B_1	1.80	-3.84	0.00	3.84
		B_2		0.00	0.00	0.00
		B_3		3.84	0.00	-3.84

Table 18

Power of the F-test under permutation, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i \neq 0]$, on A and AB for the 3 x 3 randomized block design with 6 observations per cell. Results are reported on basal responses for various combinations of intra-block variance patterns and values of kurtosis. Nominal power set at .80 for $\alpha = .05$.

				Nominal Power		Kurtosis (β_2)					
						3.0		5.0		7.0	
Variance	Q	Effect	Error	α .05	.01	.05	.01	.05	.01	.05	.01
I	.67	A	AB	.80	.35	.739	.306	.764	.334	.715	.287
		A	AB*	.80	.35	.847	.207	.811	.176	.809	.201
		A	S(AB)	.80	.56	.789	.554	.805	.565	.816	.583
		AB	S(AB)	.80	.57	.838	.595	.815	.564	.821	.618
II	1.12	A	AB	.80	.35	.761	.378	.761	.371	.714	.355
		A	AB*	.80	.35	.788	.252	.760	.231	.746	.239
		A	S(AB)	.80	.56	.790	.532	.804	.573	.807	.557
		AB	S(AB)	.80	.57	.831	.571	.814	.540	.844	.560
III	3.37	A	AB	.80	.35	.757	.388	.800	.407	.740	.381
		A	AB*	.80	.35	.764	.304	.749	.280	.712	.284
		A	S(AB)	.80	.56	.783	.533	.815	.562	.798	.538
		AB	S(AB)	.80	.57	.765	.542	.740	.505	.797	.517

* AB interaction was non-null for this run.

Table 19

Size of the F_A-test under sampling from multivariate normal populations for the repeated measures design with three levels of an independent dimension (A), four levels of a dependent dimension (B), and five and fifteen observations per level of A.

Set	Variance, σ_{jj}				Correlation, $\rho_{jj'}$						Empirical proportions: $P[F_A > F_{1-\alpha}(2,12) \mid \alpha_1 = 0]$				Empirical Proportions: $P[F_A > F_{1-\alpha}(2,42) \mid \alpha_1 = 0]$			
											5 Subjects per A level				15 Subjects per A level			
	j: 1	2	3	4	jj':12	13	14	23	24	34	α: .10	.05	.025	.01	.10	.05	.025	.01
11	100	100	100	100	.20	.20	.20	.20	.20	.20	.108	.057	.027	.014	.117	.059	.030	.011
21	100	400	900	1600	.20	.20	.20	.20	.20	.20	.101	.048	.027	.008	.106	.062	.030	.015
31	100	225	400	900	.20	.20	.20	.20	.20	.20	.090	.050	.026	.012	.093	.047	.026	.013
12	100	100	100	100	.80	.80	.80	.80	.80	.80	.094	.052	.026	.012	.115	.052	.023	.009
22	100	400	900	1600	.80	.80	.80	.80	.80	.80	.130	.063	.034	.015	.093	.047	.020	.011
32	100	225	400	900	.80	.80	.80	.80	.80	.80	.102	.040	.015	.005	.104	.049	.016	.006
13	100	100	100	100	.50	.25	.00	.25	.00	.00	.113	.055	.025	.006	.099	.040	.019	.009
23	100	400	900	1600	.50	.25	.00	.25	.00	.00	.083	.042	.022	.016	.108	.051	.023	.005
33	100	225	400	900	.50	.25	.00	.25	.00	.00	.105	.047	.023	.011	.091	.043	.015	.005
14	100	100	100	100	.50	.00	-.50	.50	.00	.50	.097	.052	.027	.008	.111	.056	.029	.015
24	100	400	900	1600	.50	.00	-.50	.50	.00	.50	.088	.042	.023	.006	.109	.054	.029	.013
34	100	225	400	900	.50	.00	-.50	.50	.00	.50	.091	.047	.028	.010	.089	.049	.025	.010
15	100	100	100	100	.80	.50	.20	.80	.50	.80	.104	.056	.033	.015	.102	.054	.032	.012
25	100	400	900	1600	.80	.50	.20	.80	.50	.80	.098	.047	.025	.014	.094	.056	.028	.011
35	100	225	400	900	.80	.50	.20	.80	.50	.80	.090	.052	.022	.006	.099	.062	.034	.013

Table 20

Size of the F_B- test under sampling from multivariate normal populations for the repeated measures design with three levels of an independent dimension (A), four levels of a dependent dimension (B), and five and fifteen observations per level of A.

Empirical proportions:
$P[F_B > F_{1-\alpha}(3\varepsilon, 36\varepsilon) | \beta_j = 0]$

Empirical proportions:
$P[F_B > F_{1-\alpha}(3\varepsilon, 126\varepsilon) | \beta_j = 0]$

		5 Subjects per A level												15 Subjects per A level											
Set	ε	Unadjusted test				ε-Adjusted test				Conservative Test				Unadjusted test				ε-Adjusted test				Conservative test			
		α:.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
11	1.00	.105	.046	.026	.010	.105	.046	.026	.010	.032	.007	.001	.000	.110	.065	.033	.015	.110	.065	.033	.015	.051	.015	.001	.000
21	.74	.112	.059	.030	.014	.084	.033	.014	.005	.041	.010	.004	.000	.103	.065	.034	.018	.084	.043	.019	.010	.056	.017	.004	.000
31	.72	.130	.067	.043	.023	.095	.050	.026	.011	.054	.019	.004	.000	.099	.056	.032	.019	.069	.035	.020	.009	.047	.018	.005	.001
12	1.00	.099	.051	.028	.010	.099	.051	.028	.010	.035	.005	.000	.000	.105	.058	.027	.009	.105	.058	.027	.009	.044	.006	.001	.000
22	.62	.127	.073	.049	.030	.094	.052	.030	.012	.060	.024	.009	.001	.117	.073	.047	.027	.103	.057	.029	.012	.065	.024	.010	.003
32	.60	.102	.077	.050	.032	.085	.053	.031	.009	.063	.024	.006	.002	.118	.083	.049	.023	.096	.053	.024	.014	.068	.020	.009	.002
13	.90	.110	.056	.031	.011	.106	.055	.028	.009	.040	.005	.001	.000	.107	.062	.037	.013	.107	.061	.036	.012	.052	.010	.002	.000
23	.64	.121	.070	.041	.020	.094	.047	.020	.012	.055	.016	.005	.001	.134	.086	.053	.028	.108	.058	.029	.013	.079	.026	.011	.002
33	.62	.107	.065	.035	.018	.081	.038	.018	.009	.050	.015	.004	.000	.120	.074	.040	.018	.098	.045	.019	.008	.063	.015	.003	.001
14	.64	.111	.064	.031	.015	.083	.035	.015	.007	.041	.014	.004	.000	.107	.072	.048	.030	.088	.052	.032	.020	.064	.027	.013	.003
24	.59	.133	.084	.053	.025	.104	.054	.024	.014	.061	.021	.006	.001	.133	.084	.056	.033	.106	.060	.033	.013	.073	.029	.007	.001
34	.55	.135	.092	.063	.043	.105	.063	.041	.025	.073	.036	.021	.006	.117	.083	.053	.028	.101	.056	.028	.016	.073	.024	.015	.005
15	.53	.117	.073	.045	.029	.090	.044	.024	.010	.053	.021	.008	.002	.110	.079	.051	.024	.097	.053	.025	.010	.070	.023	.007	.001
25	.49	.144	.101	.060	.038	.088	.037	.015	.005	.079	.035	.010	.004	.135	.092	.060	.033	.081	.033	.015	.006	.081	.030	.013	.006
35	.45	.153	.105	.071	.056	.096	.053	.022	.006	.087	.042	.019	.006	.146	.095	.067	.037	.087	.036	.017	.006	.087	.034	.017	.006

Table 21

Size of the F_{AB}-test under sampling from multivariate normal populations for the repeated measures design with three levels of an independent dimension (A), four levels of a dependent dimension (B), and five and fifteen observations per level of A.

		Empirical porportions: $P[F_{AB} > F_{1-\alpha}(6\varepsilon, 36\varepsilon) \mid (\alpha\beta)_{ij} = 0]$												Empirical proportions: $P[F_{AB} > F_{1-\alpha}(6\varepsilon, 126\varepsilon) \mid (\alpha\beta)_{ij} = 0]$											
		5 Subjects per A level												15 Subjects per A level											
Set	ε	Unadjusted test				ε-Adjusted test				Conservative test				Unadjusted test				ε -Adjusted test				Conservative test			
		α:.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
11	1.00	.106	.052	.028	.014	.106	.052	.028	.014	.028	.004	.000	.000	.113	.054	.023	.007	.113	.054	.023	.007	.028	.005	.000	.000
21	.74	.132	.075	.054	.031	.093	.055	.031	.008	.053	.014	.003	.000	.124	.066	.045	.027	.094	.049	.028	.011	.051	.019	.005	.000
31	.72	.128	.078	.045	.028	.100	.047	.028	.013	.044	.016	.004	.001	.133	.088	.054	.032	.111	.055	.032	.014	.059	.021	.009	.001
12	1.00	.101	.044	.024	.006	.101	.044	.024	.006	.022	.004	.000	.000	.109	.050	.025	.005	.109	.050	.025	.005	.029	.003	.001	.000
22	.62	.155	.098	.058	.028	.116	.055	.026	.015	.056	.022	.007	.001	.132	.081	.053	.033	.101	.055	.033	.011	.056	.022	.006	.000
32	.60	.136	.088	.057	.031	.103	.050	.030	.014	.054	.018	.006	.000	.147	.101	.064	.037	.116	.067	.036	.016	.072	.026	.006	.002
13	.90	.111	.065	.039	.012	.100	.056	.029	.009	.038	.006	.001	.000	.118	.062	.034	.017	.104	.052	.029	.011	.037	.009	.002	.001
23	.64	.136	.074	.045	.025	.094	.045	.024	.013	.043	.017	.007	.001	.110	.072	.044	.019	.088	.044	.019	.009	.050	.015	.005	.004
33	.62	.138	.084	.057	.034	.104	.055	.031	.012	.055	.020	.005	.000	.136	.085	.057	.033	.101	.057	.033	.012	.062	.016	.006	.002
14	.64	.137	.078	.048	.022	.094	.047	.022	.014	.047	.014	.006	.001	.123	.075	.052	.026	.096	.055	.026	.011	.058	.016	.005	.003
24	.59	.140	.098	.061	.032	.105	.058	.029	.018	.059	.021	.008	.001	.144	.087	.059	.037	.105	.060	.036	.019	.063	.029	.013	.001
34	.55	.140	.092	.057	.033	.090	.040	.021	.009	.055	.021	.005	.003	.143	.101	.066	.044	.103	.051	.028	.012	.074	.031	.012	.003
15	.53	.116	.079	.055	.035	.076	.040	.016	.006	.055	.016	.006	.001	.161	.092	.067	.033	.092	.044	.020	.007	.074	.023	.007	.007
25	.49	.140	.093	.058	.036	.084	.040	.027	.011	.056	.028	.011	.003	.151	.104	.069	.042	.104	.051	.032	.015	.075	.035	.015	.005
35	.45	.152	.104	.073	.044	.094	.048	.026	.009	.069	.027	.009	.004	.145	.115	.079	.061	.112	.068	.041	.019	.086	.050	.022	.009

Table 22

Means and variances of $\hat{\varepsilon}$

obtained on 1000 samples for 5 and 15 subjects per level of A

		5 Subjects		15 Subjects	
Set	ε	$\bar{\hat{\varepsilon}}$	$S^2_{\hat{\varepsilon}}$	$\bar{\hat{\varepsilon}}$	$S^2_{\hat{\varepsilon}}$
11	1.0000	.8239	.006497	.9324	.001461
21	.7357	.6684	.009289	.7147	.003835
31	.7246	.6712	.009853	.7073	.004419
12	1.0000	.8262	.006438	.9338	.001400
22	.6238	.5923	.008557	.6154	.004084
32	.5967	.5814	.008882	.5908	.003746
13	.9009	.7750	.008585	.8568	.003331
23	.6351	.5992	.007463	.6196	.003695
33	.6172	.5885	.008354	.6096	.003339
14	.6410	.6146	.010475	.6334	.004396
24	.5934	.5726	.007981	.5887	.003554
34	.5451	.5334	.007413	.5424	.002582
15	.5333	.5306	.006581	.5337	.002452
25	.4916	.4926	.004906	.4909	.001566
25	.4518	.4565	.003538	.4531	.001122

Table 23

Power of F_B and F_{AB} tests under sampling
from multivariate normal populations for five subjects in each level of A,
Nominal power = .80 and .60 for test size of .05.

				Empirical proportions: $P[F_B > F_{.95}(3\varepsilon, 36\varepsilon) \mid \beta_j \neq 0]$			Empirical proportions: $P[F_{AB} > F_{.95}(6\varepsilon, 36\varepsilon) \mid (\alpha\beta)_{ij} \neq 0]$		
		Set	ε	Unadj	ε-Adj	Cons	Unadj	ε-Adj	Cons
Power	**.60**	11*	1.00	.600	.600	.291	.602	.602	.235
		21	.74	.567	.473	.300	.586	.498	.243
		31	.72	.554	.476	.274	.578	.479	.253
		15	.53	.516	.435	.308	.567	.432	.309
		25	.49	.520	.330	.307	.542	.390	.301
		35	.45	.508	.330	.303	.530	.375	.289
Power	**.80**	11	1.00	.797	.797	.503	.816	.816	.456
		21	.74	.761	.683	.481	.784	.694	.440
		31	.72	.776	.695	.490	.797	.697	.450
		15	.53	.697	.629	.489	.697	.550	.428
		25	.49	.653	.487	.457	.706	.553	.437
		35	.45	.683	.501	.474	.701	.532	.432

*** Non-null mean vectors used for set** 11

		Power = .60					Power = .80				
Non-null Effect	Level of A	ϕ^2	B_1	B_2	B_3	B_4	ϕ^2	B_1	B_2	B_3	B_4
B	Each A_i	2.02	-4.38	-1.46	1.46	4.38	3.03	-5.40	-1.80	1.80	5.40
A x B	A_1	1.59	-6.33	-2.11	2.11	6.33	2.37	-7.74	-2.58	2.58	7.74
	A_2		0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00
	A_3		6.33	2.11	-2.11	-6.33		7.74	2.58	-2.58	-7.74

Table 24

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = 0]$, for the repeated measures design with three levels of an independent dimension (factor A), four levels of a dependent dimension (factor B) and five observations per level of A. Results are reported for basal responses with various combinations of intra-subject variance based on dependent responses and inter-subject kurtosis based on subject means.

Intra-subject variance **	Q	Effect	Inter-subject kurtosis (β_2)* 3.0 α .10	.05	.025	.01	6.0 .10	.05	.025	.01
I	0.96	A	.104	.048	.021	.002	.030	.000	.000	.000
		B	.103	.053	.030	.013	.114	.067	.030	.013
		AB	.104	.056	.030	.014	.111	.060	.028	.014
II	1.07	A	.095	.042	.014	.006	.045	.002	.000	.000
		B	.089	.037	.016	.004	.102	.039	.019	.003
		AB	.090	.040	.016	.005	.093	.047	.023	.008
III	1.29	A	.088	.041	.015	.003	.050	.000	.000	.000
		B	.069	.021	.007	.001	.078	.033	.013	.003
		AB	.066	.028	.010	.001	.071	.025	.014	.003
IV	1.40	A	.111	.054	.032	.009	.034	.000	.000	.000
		B	.080	.020	.007	.000	.074	.021	.006	.000
		AB	.065	.027	.006	.000	.066	.024	.008	.002

* Skewness level is fixed at $\beta_1 = 0.0$.

** Intra-subject variance specifications. Subject variances (biased) are as follows:

Specification	Subject Variances 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0	798.0
II	31.0	43.4	60.7	85.0	119.0	166.6	233.3	326.5	457.2	640.8	896.0	1254.4	1756.2	2458.7	3442.1
III	1.0	1.9	3.4	6.3	11.7	21.7	40.1	74.2	137.2	253.8	469.6	868.7	1607.2	2973.3	5500.5
IV	0.2	0.4	0.9	1.8	3.8	7.9	16.6	34.8	73.1	153.5	322.4	677.1	1422.0	2986.2	6271.0

Table 25

Power of the F-test under permutation, $P\left[F_t \geq F_{1-\alpha}(n_1,n_2) \middle| T_i \neq 0\right]$, for the repeated measures design with three levels of an independent dimension (factor A), four levels of a dependent dimension (factor B) and five observations per level of A. Results reported for basal responses with various combinations of intra-subject variance and inter-subject kurtosis. Nominal power for factor B and AB interaction set at .60 and .80 for α = .05

			Nominal Power		Kurtosis (β_2)			
					3.0		6.0	
Variance	C_i	Effect	α .05	.01	.05	.01	.05	.01
I	0.96	B	.60	.33	.593*	.308	.593	.314
			.80	.55	.794	.557	.795	.533
		AB	.60	.34	.577	.296	.609	.317
			.80	.56	.809	.546	.801	.540
II	1.07	B	.60	.33	.629	.342	.586	.327
			.80	.55	.818	.572	.796	.547
		AB	.60	.34	.615	.328	.610	.332
			.80	.56	.822	.572	.819	.562
III	1.29	B	.60	.33	.622	.345	.596	.333
			.80	.55	.796	.557	.801	.581
		AB	.60	.34	.619	.311	.608	.292
			.80	.56	.831	.573	.837	.600
IV	1.40	B	.60	.33	.579	.319	.614	.347
			.80	.55	.796	.543	.772	.516
		AB	.60	.34	.608	.311	.594	.318
			.80	.56	.843	.572	.806	.558

* Non-null treatment effects used for the repeated measures design for data with kurtosis = 3.0 and "variance I" specification

Non-null Effect	Level of A	Power = .60					Power = .80				
		ϕ^2	β_1	β_2	β_3	β_4	ϕ^2	β_1	β_2	β_3	β_4
B	Each A_i	2.02	-16.02	-5.34	5.34	16.02	3.03	-19.64	-6.55	6.55	19.64
AB	A_1	1.59	-23.04	-7.68	7.68	23.04	2.36	-28.15	-9.38	9.38	28.15
	A_2		0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00
	A_3		23.04	7.68	-7.68	23.04		28.15	9.38	-9.38	28.15

Table 26

Size of permutation test, $P\left[F_t > F_{1-\alpha}(n_1, n_2) \mid T_i = 0\right]$, for the split-plot design with factor A at 3 levels (whole plots), factor B at 4 levels (split plots) and 5 replications. Results are reported for basal responses with various combinations of intra-plot and intra split-plot variability.

Intra split- ** plot variance Q_B		Effect	Intrawhole-plot variance *											
			1				2				3			
			Q_A 0.96				1.29				1.40			
			α .10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
I	.80	A	.112	.073	.040	.024	.086	.023	.001	.000	.041	.000	.000	.000
		B	.107	.051	.026	.020	.082	.039	.022	.011	.089	.043	.014	.006
		AB	.096	.049	.024	.006	.109	.064	.032	.010	.108	.051	.017	.008
II	1.41	A	.105	.062	.034	.023	.055	.006	.000	.000	.069	.000	.000	.000
		B	.015	.004	.001	.000	.021	.004	.000	.000	.026	.004	.001	.001
		AB	.016	.000	.000	.000	.016	.004	.002	.001	.008	.000	.000	.000
III	1.81	A	.108	.063	.050	.020	.083	.035	.012	.000	.025	.000	.000	.000
		B	.005	.000	.000	.000	.007	.001	.000	.000	.010	.001	.000	.000
		AB	.002	.000	.000	.000	.001	.000	.000	.000	.005	.000	.000	.000

* Intra-whole-plot variances correspond to variance specifications I, III and IV of previous repeated measures design considered.

** Intra split-plot variances correspond to those specified as variances I, II and III in the 3 x 5 randomized block design with one observation per cell.

Table 27

Size of permutation test, $P[F_t > F_{1-\alpha}(n_1,n_2) | T_i = 0]$, for a two factor completely randomized repeated measures experiment with factor A at three levels, factor B at four levels and five replications. Results are reported for the nine sets of basal responses used in split plot design. *Q, the index of intra-block (replication) variability, has been included as a footnote.

Effect	α.10	.05	.025	.01	.10	.05	.025	.01	.10	.05	.025	.01
A	.093	.048	.024	.011	.043	.010	.003	.000	.048	.013	.004	.000
B	.095	.048	.026	.009	.042	.010	.001	.001	.046	.015	.006	.001
AB	.110	.064	.034	.015	.038	.009	.001	.000	.023	.008	.003	.001
A	.110	.054	.022	.010	.017	.002	.001	.001	.019	.002	.000	.000
B	.101	.052	.027	.011	.025	.003	.000	.000	.015	.002	.000	.000
AB	.119	.057	.027	.012	.018	.004	.001	.000	.008	.001	.001	.000
A	.107	.061	.033	.017	.030	.005	.000	.000	.012	.002	.000	.000
B	.121	.050	.036	.011	.019	.004	.000	.000	.016	.000	.000	.000
AB	.126	.066	.034	.019	.015	.001	.000	.000	.007	.001	.001	.000

*Values of Q, index of intra-block variability, are given below.

.98	2.13	3.00
.97	2.52	3.06
.96	2.74	3.46

APPENDIX B, COMPUTER PROGRAM MANUAL FOR PERMUTATION PROGRAM FBROC

INFORMATION MANUAL

The [illegible] Program for

ANALYSES OF EXPERIMENTAL DESIGNS BY MEANS OF [illegible]

The purpose of this program is to provide a means by which the [illegible] can obtain the empirical distribution of the variance ratios of a wide class of experimental designs by means of [illegible] techniques. The program allows the user to specify the particular experimental design and the appropriate analysis of variance in a conventional manner using the symbolic degrees of freedom information. The manner in which the observations are to be permuted can also be specified symbolically in a convenient form. The group of [illegible] [illegible] up to 1,000 samples generated at random [illegible]. The frequency distribution is constructed where the [illegible] [illegible] points of the appropriate theoretical F distribution. [illegible] options are available to the user which allow different [illegible] the number of class intervals. The output is to the printer.

General Description

The general flow of the program proceeds as described in the paragraphs below. The program is read in either from magnetic tape or a FORTRAN card deck into computer memory. The initial subroutine of the program reads in the problem description card which contains a description of the number of F-ratios, the number of error terms, the number of [illegible] cards, etc. The next subroutine of the program reads in the analysis of variance control cards, one of which describes the dimensionality of the analysis of variance and the others containing the control words for the specific analysis of variance. The program then [illegible] describing

INFORMATION MANUAL

The Baker-Collier Program for

ANALYSIS OF EXPERIMENTAL DESIGNS BY MEANS OF RANDOMIZATION (FBROC)

The purpose of this program is to provide a means by which an investigator can create the empirical distributions of the variance ratios of a wide class of experimental designs by means of Monte Carlo techniques. The program allows the user to specify the particular experimental design and the appropriate analysis of variance in a conventional manner using the symbolic degrees of freedom. In addition, the manner in which the observations are to be permuted can also be specified symbolically in a convenient form. The program system will analyze up to 1,000 samples generated by permuting the original data. A frequency distribution is constructed where the class intervals are based on specified points of the appropriate theoretical F-distribution. Various options are available to the user which allow different specifications on the number of class intervals. The output is to the printer.

General Description

The general flow of the program proceeds as described in the paragraph below. The program is read in either from magnetic tape or a FORTRAN card deck into computer memory. The initial subroutine of the program reads in the problem description card which contains a description of the number of F-ratios, the number of error terms, the number of randomization parameter cards, etc. The next subroutine of the program reads in the analysis of variance control cards, one of which describes the dimensionality of the analysis of variance and the others containing the control words for the specific analysis of variance. The program then reads in cards describing the error terms and establishes the proper calculations for the error terms desired by the user. The variance ratio calculations are then prestored from information provided on variance ratio description cards. The program then reads in a description of the randomization patterns required to permute the data according to the design of interest and will establish the proper permutation procedures within the computer program. The data and any treatment effect terms are read into computer memory along with a post-processing card which contains information concerning the class intervals of the resulting empirical distributions. Now the program is ready to proceed with analysis.

Initially, the original data are processed through the analysis of variance program without any permutations to provide the error terms and variance ratios for the original data. The second segment of the program will take the original data, permute it according to the randomization patterns, compute the desired variance ratios and build up a frequency distribution for each variance ratio. That is, a counter for the appropriate interval in the frequency distribution is advanced for each variance ratio. This process is repeated until the desired number of samples have been generated and analyzed. This program can analyze up to ten variance ratios. The frequency distributions are constructed using theoretical F points as interval boundaries, twelve F points being "built in" with up to eight optional in addition to these twelve.

A further option allows the user to specify that he desires N (where $20 \leq N \leq 50$) class intervals instead of the implicit twelve obtained under standard conditions. If this option is exercised, there may actually be up to N + 3 class intervals since the internal class boundaries are N - 1 equally spaced percentiles and P_{90}, P_{95} and P_{99}.

Capabilities and limitations of the Program

1. Randomization patterns and analysis of variance can be specified for a variable possessing up to 8 subscripts.
2. Only one variable can be employed.
3. Up to 20 stages of randomization can be specified for any given design.
4. All cells of the ANOVA design must contain the same number of observations.
5. The product of the number of levels of the indices must be less than 2500.
6. The number of ANOVA control words cannot exceed 100.
7. The number of variance ratios specified cannot exceed 10.
8. The number of error terms specified cannot exceed 20.
9. The number of sums of squares pooled to form an error term cannot exceed 20.
10. The value -9999.9999 cannot be used as data or result from adding a treatment effect term to data.
11. Embedded blanks are not permitted within any of the control cards.
12. The number of samples generated in a computer run is $\leq$ 1000.
13. All input cards contain a three-character code in columns 1 through 3 to identify their type.
14. A code word for the total sums of squares is not permitted in the ANOVA specification.

Use of the System

Randomization Specification

Terminology

BLOCKING	The data is separated into groups within which it is manipulated separately. Across block data transfers do not occur.
PERMUTE	An array of observations are re-arranged in a random fashion.
SET	An array of data which is treated as a logical unit. For example, all the scores for a given person.
STAGES	In many designs more than one type of permutation occurs. In this case each type of permutation performed is specified and performed separately.

OBSERVATION	An observation is described by a capital letter accompanied by up to 8 subscripts; for example, X_{ijkl}.

Operation Codes for Specifying Randomization

*	Signifies blocking. The array is separated into blocks each of which is processed separately. For example, *I means there are I blocks.
$	Means permute over. The observations to the left of the $ are permuted over the cells on the right. For example, M$M means the M observations are permuted into M cells. Note: This symbol cannot appear in column 80 of a card.
( )	Signifies set. The letter within the parentheses defines the size of a set. For example, (M)$L means sets of size M are permuted over the L cells with the M observarions constituting a logical group.
+	Signifies then. Separates the specifications for the various stages of permutation.
.	Means end. Specifies the end of a randomization pattern on computer input.

Operation Code Rules for Randomization Specifications

Rule 1 The permutations to be performed must be specified one stage at a time. If multiple permutations are necessary, each stage is described in full. For example, in the split-plot designs, M$M*IJKL + (M)$JKL*I, specifies two levels of randomization. Left of the + the M observations are permuted into M cells for each of the IJKL blocks of data. To the right of the + the previously permuted M observations are treated as a unit and permuted over the JKL cells within I blocks. The stages are performed from left to right.

Rule 2 The specification of a set must be followed by a permutation code. For example, (M)$JKL. The set specified by (M) must be followed by the $ mark indicating it is to be permuted over the JKL cells. This implies that parentheses are not used for grouping other than to define a set which is treated as a logical unit prior to the occurrence of a $ operation code.

Rule 3 The specification of a permutation must contain letters to the right and left of a $ code which are contiguous alphabetically. For example

(M)$JKL, M and L are contiguous. Such things and (M)$I are not allowed.

Rule 4 The letters IJKLMNOP must be used as subscripts with I being the left-most proceeding to the right.

Rule 5 A maximum of 20 stages of randomization are allowed.

Rule 6 Multiple appearances of a given operation code within a stage of randomization is not allowed. For example, MML is illegal as is M$M*K$IJ.

Examples of Randomization Patterns

Completely randomized design. IJKLM$IJKLM. The IJKLM observations are permuted over IJKLM cells. The result being the observations are completely randomized over the corresponding array of cells.

Randomized block. KLM$KLM*IJ. The KLM observations are permuted over KLM cells for each of the IJ blocks.

Split-plot. M$M*IJKL + (M)$JKL*I. The M observations within each of IJKL blocks are permuted into M cells. Then the sets of M scores are randomly assigned to JKL cells separately for each of I blocks.

Repeated measures. LM$LM*IJK + (LM)$JK*I. The LM observations are permuted over LM cells within each of IJK units. Then the sets of LM scores are permuted over the JK cells separately from each of I blocks.

Analysis of Variance Specifications

The program used is the University of Wisconsin ANOVA program with modifications. This program will compute the sum of squares, mean squares, variance ratios, and degrees of freedom which appear in analysis of variance tables. The routine will perform the calculations required for any balanced fully replicated or nested design. Fractionally replicated designs such as Latin squares and unbalanced designs such as missing data problems or proportional subclass problems cannot be handled. The routine computes only those entries of an analysis of variance table which are called for. The routine will handle designs with up to eight factors. The format for the control cards is specified in the section under card formats. Rather than use the binary coded words as in the original Wisconsin ANOVA program, a modification allows one to specify the design using the letters of the subscripts. For example, a randomized block design with K observations per cell (X_{ijk}) would be specified by

Code Word	Source of Variation
00000000	Grand Mean
I0000000	Blocks
0J000000	Treatment
IJ000000	Block X Treatment Interaction
IJK00000	Error Term

The conversion to binary code words is performed by the program.

Data cards must be in lexiographic ordering.

Specification of Error Terms

The following procedure is used so that the equivalence of an error term and the code words corresponding to particular sums of squares can be established. In general terminology, this would be indicated by E(N) = SS(IJ) + SS(JK) + ..., etc. A typical error would be set up on an error term card as follows:

E(01) = IJ000000 + 0JK00000.

where the reduced sums of squares to be pooled are specified by their corresponding code words. The error degrees of freedom will be the sum of those associated with the code words employed. Up to 20 code words can be pooled to form an error sum of squares. See the section on card formats for the exact card format specification. The error term cards need not be ordered, i.e., error one does not need to precede two in the card deck.

Specification of the Variance Ratios

The variance ratios to be calculated are established by using the code word corresponding to the mean square of interest and designating the proper error term, one variance ratio specification per card. For example, on a variance ratio card we would indicate

F(01) = I0000000/E(01).

where F(01) = designates the variance ratio to be calculated. I0000000 indicates a numerator mean square and E(01) the error term used as the denominator of the variance ratio. Again the F-ratio cards need not be ordered, i.e., F(02) could precede F(01) in the card deck.

Post Processing

The post-processing cards will specify the number of samples to be employed if less than the number generated by the previous program. Also contained on the post-processing card will be the number of additional points of the central F-distribution (if any) to be calculated over and above the standard twelve points contained in the program. If the second option is chosen, then the PST card will contain the number of intervals to be used.

Card Loading Order

1. Problem description cards
2. ANOVA control card
3. ANOVA control words (2 sets required)
4. Error term cards
5. F-ratio cards
6. Randomization pattern cards
7. Format card for data and power cards
8. Data cards, lexiographically ordered
9. Treatment effects, lexiographically ordered, if any
10. Post-processor control cards

Input Card Layouts

A. Randomization and Analysis Section

1. Problem description cards

Columns 1 through 3	Punched PDC to indicate type of card
Columns 4 through 5	The number of error term cards
Columns 6 through 8	Number of F-ratio cards, ≤ 10
Columns 9 through 11	The number of randomization pattern cards, $N \leq 5$
Column 12	Treatment effects cards present? Yes, a non-zero number; No, blank or zero
Columns 13 through 16	The number of samples to be generated where $N \leq 1,000$
Columns 17 through 18	Leave blank
Column 19	Punched 0 if no randomization patterns are desired, punched 1 for first 10 patterns
Columns 20 through 21	Blank
Columns 22 through 37	16 digit octal format, random number to initiate the random number generator. For best results this number should begin with one of the digits 1, 2 or 3 and end in an odd digit.
Columns 38 through 80	Blank

2. ANOVA control cards

Columns 1 through 3	Punched AVA
Column 4	Number of indices ≤ 8
Columns 5 through 8	Punched 0001
Columns 8 through 12	Number of control words
Columns 13 through 16	Number of levels of first index, I
Columns 17 through 20	Number of levels of second index, J
Columns 21 through 24	Number of levels of third index, K
Columns 25 through 28	Number of levels of fourth index, L
Columns 29 through 32	Number of levels of fifth index, M

Etc.	Up to 8 indices
Columns 49 through 80	Name of problem, any arbitrary label of up to 32 characters

3. ANOVA code words - 2 sets required

Columns 1 through 3	Punched ACW
Column 4	Blank
Columns 5 through 12	First control word, 00000000
Columns 13 through 20	Second control word
Columns 21 through 28	Third control word, etc.
Columns 77 through 80	Do not use
	If any additional cards needed, use columns 5-76 with no additional specification necessary on the card.

4. Error term cards

Columns 1 through 3	Punched ETC
Columns 4 through 5	Blank
Column 6	Continuation card if punched 1, blank if first card
Columns 7 through 80	Error term specifications

Each specification must be begun on a separate card although it may continue over to additional continuation cards. If an error term specification exceeds a card, it must end with a + and the next card must contain a 1 punch in column 6 with the first letter of the next code appearing in column 13. The last card <u>must contain a period immediately following the last symbol</u>. For example,

```
E(02)=I0000000+IJK00000+
      IK000000+IJK00000.
```

Note: <u>Blanks are not allowed</u> within error term specifications.

5. F-ratio cards

One F-ratio is specified per card.

Columns 1 through 3	Punched FRC to specify card type
Columns 7 through 80	The F-ratio specification. For example, F(01)=IJK00000/E(02)

6. Randomization pattern cards

Columns 1 through 3	Punched RPC
Columns 7 through 80	is the randomization pattern. If necessary to continue to the next card, punch a 1 in column 6 and continue. Begin in column 7;up to 5 continuation cards will be allowed, with string terminated by a period. For example, M$M*IJKL+(M)$JK*I. Note: no $ in Column 80 allowed.

7. Format card for the data and treatment effect cards

The format cards specify the arrangement of data on the data cards under the following restrictions:

a. The data must be in either E or F format.

b. The format specifies only one observation per card.

c. The first three columns of the card are punched FMT to designate a a format card.

d. The first variable on the data card must be specified by an A3 field and the second variable by an integer field which contains the sequence number for the particular observation, first sequence number must be > 0.

e. The format statement begins with an open parenthesis in column 9 on the format card. Example format card FMT.....(A3,I6,10X,F6.0).

8. Data Card

a. Each card must contain a single observation accompanied by a sequence number which designates its position in the design. On the data card the sequence number should precede the observation and follow the card type designator DTA. For example, in a two-way layout, say 2x3x10 where I = 2 (Rows); J = 3 (Columns); and K = 10 (Subjects), we could use either of the following sets of data cards.

DTA01	X_{111}	DTA1101	$X_{1,1,1}$
		.	
		.	
		.	
		DTA1110	$X_{1,1,10}$
		DTA1201	$X_{1,2,1}$
		.	
		.	
		.	
		DTA1210	$X_{1,2,10}$
		DTA1301	$X_{1,3,1}$
		.	
		.	
		.	
DTA60	$X_{2,3,10}$	DTA1310	$X_{1,3,10}$
		.	
		.	
		.	
		Etc. for I = 2	

Note the integer number formed by the sequence number must be less than $2^{47}-1$.

b. The input cards must be placed in lexiographic order as the computer checks for monotonically increasing setsof sequence numbers. (This is slightly different from the Wisconsin analysis of variance program but is done in this manner so that the data will be insured to be in proper order before proceeding with several hours of computing.)

9. Treatment effect cards

The format of these cards must be identical to that of the data cards except that the identifier POW is punched in columns 1 through 3, and it is specified by the same format cards.

B. Post Processor

1. Control Card

Columns 1 through 3	PST
Columns 4 through 5	Blank

Columns 6 through 9	Number of samples to be used $N \leq 1000$
Column 10	Blank
Column 11	Number of additional F % points to be calculated $N \leq 6$.
Columns 12 through 15	Additional F % point, e.g., .985
Columns 16 through 19	Additional F % point
Columns 20 through 23	Additional F % point
Columns 24 through 27	Additional F % point
Columns 28 through 31	Additional F % point
Columns 32 through 35	Additional F % point
Columns 36 through 37	Number of grouping intervals to be used for frequency distribution of F's if this option is used. $20 \leq N \leq 50$.
Columns 38 through 40	Number of F-ratios selected for analysis ≤ 10.
Columns 41 through 50	1's in cols 41,42,...,40+K where K is the number of F-ratios.

OUTPUT

1. Name of Job
2. Pseudo-analysis of Variance Table for Base Data
3. Empirical F-Distributions, one per page, in order designated by FRC cards. The source of variation for the numerator as indicated on the ACW card, e.g., 10000000, is used to identify these tables. In order to appear here, the appropriate column of the PST card must be punched 1. The table headings have the following significance.

   FPP Indicate the percentile points of the theoretical F-distribution used as class interval boundaries. If columns 36, 37 and 11 on the PST card are blank, the nine deciles along with $P_{.95}$, $P_{.975}$ and $P_{.99}$ will be used. Columns 11 through 35 are used to designate how many (≤ 6) and which extra FPP points are to be used. If columns 36-37 are nonzero (between 20 and 50) then this many equally spaced (with respect ot the expected proportion of observations) intervals will be used. In addition this option always provides P_{90}, P_{95} and P_{99}. For example, if 50 is punched in columns 36-37 of the PST card, the class interval boundaries will be 0, P_2, $P_4, \ldots, P_{90}, P_{92}, P_{94}, P_{95}, P_{96}, P_{98}, P_{99}$ and ∞.

FTHEORY Gives the actual percentiles for each of the values of FPP.

FCOUNT Gives a count of the empirical variance ratios which fell into class intervals with the FTHEORY entry as the upper boundary and the next smallest one as the lower boundary. The first number gives the count between zero and the first F theory point while the last gives the count above P_{99}.

FOBS Converts the FCOUNT column into proportions.

FPCT Cumulates the proportions from the upper end of the empirical distribution to the lower.

4. On the last page, along with the last frequency distribution, will appear two columns headed DATA VECTOR and TRT VECTOR. The DATA VECTOR lists the base data in the order in which the data cards were read into computer memory. Similarly the treatment vector lists the POW cards read in (if any).
5. The next output yields average error mean squares. These are averages, over however many permutations of the data are performed, for each of the various error mean squares as specified on the ETC cards. These are included because some estimate of the population error mean square is needed when power is to be investigated. In some cases the expected value of the error mean square can be calculated directly from the base data (e.g., in a randomized block design) but in other cases this is not possible.

6. Finally, a correlation matrix of the empirical variance ratios is printed out in order to obtain a little information concerning the relationships between the variance ratios.

Error Alarms

The error alarms are all of the same general form; a message is printed describing the error, a stop instruction is printed, and the run terminates. Because the program runs for long periods of time using the information from the input cards, the sequencing of these input cards is carefully checked and improper order will termintate the run. The program also decodes a number of symbolic representations in order to specify the internal order of operations, if the input cannot be properly decoded the run will terminate. The paragraphs below describe the alarms and indicate briefly the cause of the alarm.

1

IMPROPER PROBLEM DESCRIPTION CARD IDENT

STOP 1010

The alarm is printed if the first three characters on the problem description card do not contain PDC in columns 1-3.

2

IMPROPER IDENT ON AVA CARD

STOP 1020

The analysis of variance control card did not contain AVA in columns 1-3.

3

THIS DESIGN IS TOO LARGE FOR MEMORY

STOP 1030

The product of the upper level of the indices is 4096. Check the AVA card for the values punched in the level fields.

4

READ IMPROPER ERROR TERM IDENT

STOP 1040

Columns 1-3 of an error term card did not contain ETC.

5

OP CODE NOT + NOT .

STOP 1050

A code word on an error term card was not followed by a + or a . as required. Can be caused by embedded blanks between code word and operation code.

6

ERROR TERM CODE WORD AND ANOVA CODE WORD NO MATCH

STOP 1060

A code word used to designate a sum of squares within an error term was not one described on the ANOVA control word card. The error terms use a table looking up procedure on the ANOVA control words and this alarm occurs when the table does not yield the code word desired.

7

IMPROPER IDENT OF F-RATIO CARD

STOP 1070

Columns 1-3 of an F-ratio specification card did not contain FRC.

8

F-RATIO CODE WORD NOT MATCH ANOVA CONTROL WORD

STOP 1080

A code word specifying a mean square in an F-ratio did not appear on the ANOVA control word card. This error is similar to error 6.

9

RPC IDENT ERROR

STOP 1090

Columns 1-3 of the randomization pattern card did not contain RPC.

10

RANDOMIZATION PATTERN NOT END WITH .

STOP 1100

The program looks for a period to terminate the string of randomization pattern stage specifications;if it cannot find a . this alarm is printed.

11

TRIED TO DECODE TOO MANY LEVELS

STOP 1110

The randomization pattern card is scanned for +'s and the . within the specification; if the number of stages so determined is > 20, this alarm results.

12

FIRST OP CODE NOT A $

STOP 1120

Within each stage of randomization the first operation code, excluding parentheses, must be a $ permutation code.

13

ILLEGAL SPECIFICATION OF SET AFTER $ OP CODE

STOP 1130

If a set is specified it must appear to the left of the first $ code within a stage. The program inspects for parentheses to the right of the $ and if found this alarm results.

14

LEFT AND RIGHT OF $ NOT EQUAL

STOP 1140

When a set has not been specified the program checks to ascertain if the letters to the left of the $ are the same as those between the $ and the next right most operation code. For example, K$K*IJ is legal but K$J*I is illegal.

15

ILLEGAL OP CODE

STOP 1150

An operation code appeared within a randomization specification which was not a $, *, + or .

16

ILLEGAL LETTER ___

STOP 1160

During the decoding of the randomization patterns the program detected a letter which was not I, J, K, L, M, N, O, P, or blank.

17

DID NOT FIND CODE WITHIN 20 CHARACTERS

STOP 1170

In that blanks are permitted within a randomization specification, the program scans to determine if an OP code appears within 20 successive characters; if not, too many columns have been used to specify a randomization stage.

18

DATA CARD IDENT ERROR

STOP 1180

Columns 1-3 on a data card did not contain DTA.

19

DATA CARD SEQUENCE ERROR

STOP 1200

The sequence numbers on the data cards were not in monotonic increasing order.

20

TREAT EFFECT IDENT ERROR

STOP 1200

Columns 1-3 of a treatment effect card did not contain POW.

21

TREAT EFFECT CARD SEQUENCE ERROR

STOP 1210

The sequence numbers on the treatment effect cards were not in monotonic increasing order.

APPENDIX C, PRINTOUT OF COMPUTER PROGRAMS

PROGRAM FBROC

```
      PROGRAM FBROC
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(6),KLBA(100),KDF(100),
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
 9999 READ INPUT TAPE 2,1,ID,NET,NFR,NRPC,IP,NSAMP,IX,IY,IZ,KRAND
    1 FORMAT(A3,I2,2I3,I1,I4,2X,3I1,O16)
      IF(ID-3HPDC)100,55,100
   55 CONTINUE
      NSAMP=NSAMP-1
      DO 300 K=1,NET
  300 CUMEMSQ(K)=0.0
      KOUNT =0
      DO 225 I=1,NFR
      FSUM(I)=0.0
      DO 225 J=1,NFR
  225 FCP(I,J)=0.0
      CALL ANOVAA
      CALL ETSET
      CALL FRSET
      CALL DECODE
      CALL ANOVAB
      ID=(3HTPH)
      DO 7 I=1,LCMAX
    7 FRATIO(I,1) =DATA(I)
      IF(IP) 8,9,8
    8 DO 10 I=1,LCMAX
   10 FRATIO(I,1) =FRATIO(I,1) + POWER(I)
    9 CALL ANOVAC
C     THIS BEGINNING OF SEGMENT 2
   12 KOUNT = KOUNT + 1
      DO 11 I= 1,LCMAX
   11 FRATIO(I,1) = DATA(I)
      CALL PERMUTE
      IF(IP) 13,14,13
   13 DO 15 I = 1,LCMAX
   15 FRATIO(I,1) = FRATIO(I,1) + POWER(I)
C     ADD TREATMENT EFFECT TO RANDOMIZED DATA
   14 CALL ANOVAC
      DO 301 K=1,NET
  301 CUMEMSQ(K)=CUMEMSQ(K)+EMSQ(K)
      IF (KOUNT-NSAMP) 12,16,16
   16 PRINT 7000,NAME
 7000 FORMAT(4H0RUN,4A8)
      PRINT 7001
 7001 FORMAT(32H0DATA VECTOR           TRT VECTOR)
      DO7002 I=1,LCMAX
 7002 PRINT 7003,DATA(I),POWER(I)
 7003 FORMAT(F10.4,10X,F10.4)
   17 FORMAT(4X,O16)
      DO 7005 K=1,NET
 7005 CUMEMSQ(K)=CUMEMSQ(K)/NSAMP
      PRINT 7006
 7006 FORMAT(27H0AVERAGE ERROR MEAN SQUARES)
      PRINT 7007,(CUMEMSQ(K),K=1,NET)
 7007 FORMAT(10F11.4)
      FNUM=FLOATF(NSAMP+1)
      DO 7008 I=1,NFR
      DO 7008 J=1,NFR
 7008 FCORR(I,J)=(FNUM*FCP(I,J)-FSUM(I)*FSUM(J))/SQRTF((FNUM*FCP(I,I)
     1-FSUM(I)**2)*(FNUM*FCP(J,J)-FSUM(J)**2))
      PRINT 7009
 7009 FORMAT(16H0F CORREL MATRIX)
      DO 7010 I=1,NFR
 7010 PRINT 7011,(FCORR(I,J),J=1,NFR)
 7011 FORMAT(10F10.4)
      GO TO 9999
  100 END
      SUBROUTINE POST
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(1025,8),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100),
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
       DIMENSION CUMEMSQ(100)
       DIMENSION FTP(11,53,2)
       DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
       COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
       COMMON CUMEMSQ,NVOID
       COMMON FTP
       COMMON FCORR,FCP,FSUM
       IF (KOUNT) 150,150,151
  150 READ INPUT TAPE 2,1,ID,IP1,IP2,NUMSP,IP4,(FPP(I),I=13,18),INT,
     1IP6,(IDXFR(J),J=1,10)
    1 FORMAT(A3,2I1,I4,1X,I1,6F4.4,I2,I3,10I1)
      IF(ID-3HPST)2,3,2
    2 PRINT 4
    4 FORMAT (25H.POST CONTROL IDENT ERROR)
    5 STOP 5
   28 FORMAT(1X,I6,3X,A8,5E13.4)
    3 NUMSP=NUMSP-1
  252 FORMAT(6X,F10.4)
      IF(NUMSP-NSAMP)175,175,12
   12 PRINT 13
   13 FORMAT (45H ASKED TO ANALYZE MORE SAMPLES THAN AVAILABLE)
   14 STOP 14
  175 IF(INT)11,11,170
  170 IF(INT-50)171,171,172
  172 INT=50
  171 KKK=INT-1
      FINT=INT
      FINT=1.0/FINT
      DO 173 I=1,KKK
      FI=I
  173 FTP(11,I,1)=FI*FINT
      KKF=0
      FTP(11,KKK+1,1)=1.000
      FTP(11,KKK+2,1)=.900
      FTP(11,KKK+3,1)=.950
      FTP(11,KKK+4,1)=.990
      KKG=1
      DO 176 J=2,4
  181 IF(FTP(11,KKK+J,1)-FTP(11,KKG,1)) 178,179,180
  180 FPP(KKG+KKF)=FTP(11,KKG,1)
      KKG=KKG+1
      GO TO 181
  179 FPP(KKG+KKF)=FTP(11,KKG,1)
      KKG=KKG+1
      GO TO 176
  178 FPP(KKG+KKF)=FTP(11,KKK+J,1)
      KKF=KKF+1
  176 CONTINUE
      KKK=KKK+KKF
      GO TO 174
   11 KKK=12+IP4
C     SORT F PER CENT POINTS INTO ASCENDING ORDER
      FPP(1)=.100
      FPP(2)=.200
      FPP(3)=.300
      FPP(4)=.400
      FPP(5)=.500
      FPP(6)=.600
      FPP(7)=.700
      FPP(8)=.800
      FPP(9)=.900
      FPP(10)=.950
      FPP(11)=.975
      FPP(12)=.990
   16 DO 15 I = 1,KKK
      DO 17 J = I,KKK
      IF(FPP(I)-FPP(J))17,17,18
   18 TEMP1 = FPP(I)
      FPP(I) = FPP(J)
      FPP(J) = TEMP1
   17 CONTINUE
   15 CONTINUE
  174 FTP(11,20,2)=KKK
      FTP(11,19,2)=IP6
C     CONVERT F SPECS FROM 1 AND 0 TO LOCATION SUBSCRIPTS
      IF(IP6-NFR) 19,19,20
   20 PRINT 21
   21 FORMAT (28H ASKED FOR TOO MANY F RATIOS)
   22 STOP 22
   19 J=1
```

```
      LFLST=0
      DO 123 I=1,NFR
      IF(IDXFR(I)) 24,23,24
   24 IDXFR(J) = I
      LFLST=LFLST+1
   23 J=J+1
  123 CONTINUE
      DO 156 I=1,10
      DO 156 J=1,50
  156 FTP(I,J,2) =0.0
  166 DO 49 I=1,IP6
      NI=IDXFR(I)
      DO 49 K = 1,KKK
      P = FPP(K)
      FC= 0.0
      M=DFN(NI)
      N = DFD(NI)
      CALL FDISTR(M,N,FC,P)
      FTP(I,K,1)=FC
   49 CONTINUE
  151 KKK=FTP(11,20,2)
      IP6=FTP(11,19,2)
      KA=KKK+1
      DO 34 L=1,IP6
      FTP(11,L,1)=F(L)
   34 CONTINUE
      DO 153 I=1,IP6
      DO 154 J=1,KKK
      IF(FTP(I,J,1)-FTP(11,I,1)) 154,154,155
  155 FTP(I,J,2)=FTP(I,J,2)+1.0
      GO TO 153
  154 CONTINUE
      FTP(I,KA,2)=FTP(I,KA,2)+1.0
  153 CONTINUE
      IF(NSAMP-KOUNT)157,157,100
  157 TEMP1=NSAMP+1
      DO 158 I=1,IP6
      TEMP2=1.0
      K=IFX(I)
      PRINT 1101, KCCW(K)
 1101 FORMAT(31H1EMPIRICAL VARIANCE RATIO FOR   ,A8)
      PRINT 159
  159 FORMAT(1H0,19X,3HFPP,6X,7HFTHEORY,7X,6HFCOUNT,9X,4HFOBS,9X,4HFPCT
      DO 160 J=1,KKK
      M=J-1
      FTP(11,J,2)=FTP(I,J,2)/TEMP1
      IF(M)160,160,1100
 1100 TEMP2=TEMP2-FTP(11,M,2)
  160 PRINT 161,FPP(J),FTP(I,J,1),FTP(I,J,2),FTP(11,J,2),TEMP2
  161 FORMAT(10X,5F13.3)
      FTP(11,KA,2)=FTP(I,KA,2)/TEMP1
      TEMP2=TEMP2-FTP(11,KKK,2)
      PRINT 165,FTP(I,KA,2),FTP(11,KA,2),TEMP2
  165 FORMAT(36X,3F13.3)
  158 CONTINUE
  100 CONTINUE
      RETURN
      END
      SUBROUTINE FDISTR(MM,NN,F,P)
      A=MM
      A=A/2.0
      B=NN
      B=B/2.0
      X=XVSBTAF(A,B,P)
      F=B*X/((1.0-X)*A)
      END

      SUBROUTINE INCBR(A,B,X,P,XLB)
      XLB=XLGF(A)+XLGF(B)-XLGF(A+B)
      XI=1.0-X
      XH=X-0.5
      SA=A-1.0
      SB=B-1.0
      SAB=A-B
      I=0
      IF(X) 12,14,1
    1 IF(XI) 13,14,2
    2 IF(SA) 3,4,5
    3 IF(SB) 6,16,8
    4 IF(SB) 15,14,15
    5 IF(SB) 9,16,10
    6 IF(XH) 22,7,23
    7 IF(SAB) 22,14,23
    8 IF(XH) 41,23,24
    9 IF(XH) 21,22,44
   10 IF(XH) 21,11,24
   11 IF(SAB) 24,14,21
   12 P=0.0
      RETURN
   13 P=1.0
      RETURN
   14 P=X
      RETURN
   15 P=1.0-XI**B
      RETURN
   16 P=X**A
      RETURN
   21 P=BETA1F(A,B,X,XLB+LOGF(A))
      RETURN
   44 I=I+1
   22 R=EXPF(A*LOGF(X)-XLB-LOGF(A))
      IF(I) 45,46,45
   45 IF(ABSF(R)-0.00000001) 46,46,24
   46 P=BETA2F(A,B,X,R)
      RETURN
   41 I=I+1
   23 R=EXPF(B*LOGF(XI)-XLB-LOGF(B))
      IF(I) 42,43,42
   42 IF(ABSF(R)-0.00000001) 43,43,21
   43 P=1.0-BETA2F(B,A,XI,R)
      RETURN
   24 P=1.0-BETA1F(B,A,XI,XLB+LOGF(B))
      IND=4
      RETURN
      END

      FUNCTION BETA1F(S,T,X,R)
      EPS=0.000000001
      Z=X/(1.0-X)
      ZK=0.0
      SUM=0.0
      TK=EXPF(S*LOGF(Z)-(S+T-1.0)*LOGF(1.0+Z)-R)
      I=-1
      M1=0
      M2=0
      M3=XINTF(T)
      TMP=(T-S)/2.0
      IF(TMP) 3,3,1
    1 M2=XINTF(TMP)
      TMP=(T*Z-S)/(1.0+Z)
      IF(TMP) 3,3,2
    2 M1=XINTF(TMP)
    3 I=I+1
      SUM=SUM+TK
      ZK=ZK+1.0
      TK=TK*Z*(T-ZK)/(S+ZK)
   12 IF(I-M3) 4,6,6
    4 IF(I-M2) 5,5,7
    5 IF(I-M1) 3,3,9
    6 IF(ABSF(TK)/(1.0+Z)-EPS) 10,10,3
    7 IF(1.0-Z) 8,11,8
   11 IF(TK) 3,10,3
    8 IF(TK/(1.0-Z)-EPS) 10,10,3
    9 TMP=M2-I
      IF((TMP+1.0/(1.0-Z))*TK-EPS) 10,10,3
   10 BETA1F=SUM+TK
      END

      FUNCTION BETA2F(S,T,X,R)
      EPS=0.0000000001
      ZK=0.0
      SUM=0.0
      TK=R
      TM=INTF(T)
      I=-1
      M3=TM
      TMP=T-TM
      IF(TMP) 2,1,2
    1 REM=0.0
      GO TO 3
    2 REM=EXPF((S+TM)*LOGF(X)+XLGF(S+T)-XLGF(S)-XLGF(TMP)-XLGF(TM+1.0)).
     1((S+TM)*(1.0-X))
    3 SUM=SUM+TK
      I=I+1
      ZK=ZK+1.0
      CK=(S+ZK-1.0)*(T-ZK)/((S+ZK)*ZK)
      SK=-X*CK
      TK=SK*TK
   15 IF(I-M3) 4,4,6
    4 IF(1.0-ABSF(CK)) 5,5,13
    5 IF(1.0-ABSF(SK)) 3,3,7
    6 IF(ABSF(TK)/(1.0-X)-EPS) 14,14,3
    7 IF (2.0*ABSF(TK)+REM-EPS) 14,14,3
   13 IF(ABSF(TK)+REM-EPS) 14,14,3
   14 BETA2F=SUM+TK
      END

      FUNCTION XLGF(X)
    1 FORMAT(11H XLGF ARG =,E16.10)
```

```
      XLGF=0.0
      Y=1.0
      Z=X
      IF(Z.GT.0.0) 2,7
    2 IF(Z.LT.10.**300.) 3,7
    3 IF(Z.GT.4.0) 9,4
    4 IF(Z.EQ.1.0) 10,5
    5 IF(Z.EQ.2.0) 10,6
    6 Y=Y*Z
      Z=Z+1.0
      IF(Z.GT.4.0) 8,6
    7 PRINT 1,Z
      STOP
    8 XLGF=LOGF(Y)
    9 Y=LOGF(Z)
      Y=Y*(Z-0.5)-Z-XLGF
      XLGF=-1.0/Z
      Z=XLGF*XLGF
      XLGF=(((((.00129175269*Z-.000847457627)*Z+.000595238095)*Z-.000793
     1650794)*Z+.002777777778)*Z-.083333333333)*XLGF+.918938533204+Y
   10 END

      FUNCTION XVSBTAF(A,B,P)
      DIMENSION S(25)
      EPS=0.00000001
      SA=A-1.0
      SB=B-1.0
      SP=P
      IF(SP) 2,3,1
    1 IF(1.0-SP) 2,3,5
    2 STOP 2
    3 X=SP
    4 XVSBTAF=X
      RETURN
    5 IF(SA) 7,8,6
    6 IF(SB) 10,9,11
    7 IF(SB) 11,9,10
    8 X=1.0-(1.0-SP)**(1.0/B)
      GO TO 4
    9 X=SP**(1.0/A)
      GO TO 4
   10 X=A/(A+B)
      GO TO 12
   11 X=SA/(SA+SB)
   12 CALL INCBR(A,B,X,PP,R)
      TP=SP-PP
      IF(ABSF(TP)-EPS) 4,4,13
   13 IF(TP.LT.PP) 24,14
   24 IF(PP.LT.1.0+TP) 19,14
   14 XN=X+TP*EXPF(R-SA*LOGF(X)-SB*LOGF(1.0-X))
      IF(XN) 17,17,15
   15 IF(1.0-XN) 18,18,16
   16 X=XN
      GO TO 12
   17 X=X*SP/PP
      GO TO 12
   18 X=(X-X*SP+SP-PP)/(1.0-PP)
      GO TO 12
   19 XL1=LOGF(X)
      XL2=LOGF(1.0-X)
      SX=X
      ABSNX=1.0
      TA=-1.0
      TB=-1.0
      TAB=0.0
      PN=1.0
      DO 20 I=1,25
   20 S(I)=0.0
      S(2)=1.0
      DO 22 K=2,10
      CK=0.0
      ZK=K-1
      DO 21 I=2,K
      J=K-I+2
      ZJ=J
      S(J)=(ZJ-2.0-TA)*S(J)+SX*(TAB-ZJ+3.0)*S(J-1)
   21 CK=CK+S(J)
      TA=TA+A
      TB=TB+B
      TAB=TA+TB
      PN=TP*PN
      ABSOX=ABSNX
      XK=CK*PN*EXPF(ZK*R-TA*XL1-TB*XL2-XLGF(ZK+1.0))
      ABSNX=ABSF(XK)
   22 X=XK+X
      GO TO 12
      END
C        ANOVAA
      SUBROUTINE ANOVAA
C     JENNRICKS ANOVA HAS BEEN BROKEN UP INTO FOUR SUBROUTINES THE FIRST
C     (ANOVAA) READS IN THE CODE WORDS AND SETS UP THE ARRAYS OF DIGITS
C     AND OCTAL WORDS , THE SECOND(ANOVAB) USES THE CODE WORDS ETC TO
C     SET UP THE SUMS OF SQUARES CALCULATIONS, THE THIRD(ANOVAC) READS
C     THE DATA AND POWER TERNS IF ANY, THE FOURTH(ANOVAD) COMPUTES THE
C     COMPLETE ANOVA TABLE USING THE ERROR TERM AND F RATIOS SPECIFIED
C     ON THE INPUT CARDS
C        KARD(10)                   ANOVAY    BCD CARD IMAGE ARRAY
C        KCRD                       ANOVAY    COMPUTED NUMBER OF CARDS
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
C     READS IN CONTROL CARD,CONTROL WORDS AND SETS UP THE ARRAYS
```

```
C     INC,CWD,KCCW FOR USE BY THE ANOVA SUBROUTINES
      READ INPUT TAPE 2,1,ID,NF,NDV,NCW,(NL(I),I=2,9),(NAME(I),I=1,4)
    1 FORMAT(A3,I1,10I4,8A8)
      PRINT 1021,NAME
 1021 FORMAT(15H1FBROC PROGRAM  ,4A8////)
      IF(ID-3HAVA) 13,12,13
   13 PRINT 14
   14 FORMAT(28H  IMPROPER IDENT ON AVA CARD)
 1020 STOP 1020
C     THE ACW CARDS ARE READ FROM TAPE 2 INTO KCCW THEN WRITTEN
C     ON THE USERS OUTPUT TAPE AS A SCRATCH AND RE-READ INTO INC AS
C     THE SAME INFORMATION MUST BE READ UNDER TWO FORMATS.
   12 READ INPUT TAPE 2,
     1     4,(KCCW(I),I=1,NCW)
    4 FORMAT (4X,9A8,4X)
C     CLEAR INTEGER CODE WORD ARRAY
      DO 5 I = 1,9
      DO 5 J = 1,100
    5 INC (I,J) = 0
      READ INPUT TAPE 2,
     1     6,((INC(L,M),L=2,9),M=1,NCW)
    6 FORMAT(4X,72A1,4X)
      DO 9 J = 1,NCW
      DO 9 I = 2,9
      IF(INC(I,J)-1H0) 8,7,8
    7 INC (I,J) = 0
      GO TO 9
    8 INC (I,J) = 1
    9 CONTINUE
      CALL OCW
      NL(1)=1
    2 NF=NF+1
      LCMAX=1
      DO 35 L=1,NF
   35 LCMAX=LCMAX*NL(L)
      DO36 M=1,NCW
      I=1
      DO37 L=1,NF
   37 I=I*NL(L)**INC(L,M)
      NDF(M)=LCMAX/I
      IF(INC(1,M)) 38,38,39
   38 DO 10 L=2,NF
      IF(INC(L,M)) 41,41,10
   10 INC(L,M)=0
   21 LSQ(M)=NF
      GO TO 19
   41 LSQ(M)=L-1
      GO TO 19
   39 LSQ(M)=1
      INC(1,M)=0
   19 DO36 L=1,NF
      I=1-INC(L,M)
      DO 52 J=L,NF
   52 I=I*NL(J)**INC(J,M)
   36 INC(L,M)=1-I
      LBA(1)=1
      DO 53 M=2,NCW
   53 LBA(M)=LBA(M-1)+1-INC(1,M-1)
      DO 54 K=1,NCW
   54 KLBA(K)=LBA(K)
```

```
      LBASS=LBA(NCW)-INC(1,NCW)+1
      I1=LBASS+NCW
      IF(4096-I1) 25,27,27
   25 PRINT 33
   33 FORMAT(//39HTHIS DESIGN IS TOO LARGE FOR THE MEMORY)
 1030 STOP 1030
   27 RETURN
      END
C       ETSET
      SUBROUTINE ETSET
C     A PROGRAM TO ESTABLISH THE ARRAY OF SUBSCRIPTS TO BE USED IN THE
C     ERROR TERM CALCULATIONS, ASSUMES ARRAY KCCW IN MEMORY
C         KNTE                    ETSET     COUNT OF ERROR TERMS READ IN
C         NER                     ETSET     ERRTERM SUBSCRIPT ON INPUT CARD,
C         KECW                    ETSET     ERROR TERM CODE WORD
C         KEOP                    ETSET     OP CODE IN ERROR TERM SPECIFICAT
C         KER                     ETSET     ERROR TERM SUBSCRIPT
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
      DO 50 I=1,100
      DO 50 J=1,20
   50 IDXE(I,J) =0
    1 KNTE = 0
      K=1
    3 IF(KNTE-NET) 7,26,7
    7 READ INPUT TAPE 2,
     1    9,ID,IDC,NER,(KECW(I),KEOP(I),I = 1,7)
    9 FORMAT(A3,2X,I1,2X,I2,2X,7(A8,A1))
      IF(ID-3HETC) 10,11,10
   10 PRINT 12
   12 FORMAT (32H READ IMPROPER ERROR TERM IDENT)
 1040 STOP 1040
   11 IF(IDC-1)14,15,14
   14 KNTE = KNTE +1
      KER = NER
C     TABLE LOOK UP OF CODE WORD TO DETERMINE SUBSCRIPT
   15 I = 0
   16 K = K+1
      I = I+1
      DO 17 J=1,NCW
      IF(KECW(I)-KCCW(J))17,19,17
   19 IDXE(KER,K)=J
      IF(KEOP(I)-1H+)20,21,20
   21 IF(I-7) 16,7,16
   20 IF(KEOP(I)-1H.)22,23,22
   22 PRINT 24
   24 FORMAT (21H  OP CODE NOT + NOT .)
 1050 STOP 1050
   23 IDXE(KER,1)=K-1
      K=1
      GO TO 3
   17 CONTINUE
      PRINT 28
   28 FORMAT (50H ERROR TERM CODE WORD AND ANOVA CODE WORD NO MATCH)
 1060 STOP 1060
   26 CONTINUE
      RETURN
      END
C         FRSET
      SUBROUTINE FRSET
C     PROGRAM TO SET UP SUBSCRIPTS OF MEAN SQUARES AND ERROR TERMS
C     USED TO COMPUTE F RATIOS
C         KFX                      FRSET      SUBSCRIPT OF F RATIO
C         KNCW                     FRSET      UPR LIM ON CWD WRDS SRCHED
C         KFCW                     FRSET      MEAN SQUARE CODE WORD
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
    1 DO 12 I = 1,NFR
      READ INPUT TAPE 2,
     1     2,ID,KFX,KFCW,KEX(KFX)
    2 FORMAT (A3,5X,I2,2X,A8,3X,I2)
      IF(ID-3HFRC)10,9,10
    9 DO 3 K = 1,NCW
      IF(KFCW-KCCW(K))3,5,3
    5 IFX (KFX) = K
      GO TO 12
    3 CONTINUE
    7 PRINT 14
   14 FORMAT (48H  F RATIO CODE WORD NOT MATCH ANOVA CONTROL WORD)
 1080 STOP 2010
   12 CONTINUE
      GO TO 13
   10 PRINT 11
   11 FORMAT (31H  IMPROPER FRATIO IDENT ON CARD)
 1070 STOP 1070
   13 RETURN
      END
C     DECODE
      SUBROUTINE DECODE
C     THIS PROGRAM READS THE RANDOMIZATION PATTERNS AND ESTABLISHES
C     THE NUMBER AND SIZE OF THE SOURCES AND CELLS PLUS THE CLUSTERING
C     FLAGS FOR USE BY THE RANDOMIZATON SUBROUTINE
C         KARAC(370)       1     DECODE     ARRAY OF CHARACTERS FROM RPC
C         NSTAGE           1     DECODE     NUMBER OF RANDOMIZATION STAGES
C         NSOURCE(20)      1     DECODE     NUMBER OF SOURCES
C         KSRCSZ(20)       1     DECODE     SOURCE SIZE
C         NCELLS(20)       1     DECODE     NUMBER OF CELLS
C         KCELSZ(20)       1     DECODE     CELL SIZE
C         KCLFLG(20)       1     DECODE     CLUSTERING FLAG
C         MAX              1     SCAN       PRODUCT OF SUBSCRIPT LEVELS,MAX
C         KCOUNT           1     DECODE     COUNT OF CHARACTER LIST, TOTAL
C         KARCNT                 DECODE     TOTAL NO CHARACTERS IN R-PATTERN
C         LRBD            1      DECODE     LOWER BOUND CHARACTER INPUT LIST
C         LUPBD                  DECODE     UPPER BOUND CHARACTER INPUT LIST
C         NTLPR                  DECODE     TEMP STORAGE PRODUCT OF LETTERS
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
      LRBD = 1
      LUPBD = LRBD + 73
      DO 2 I = 1,NRPC
      READ INPUT TAPE 2,
     1    303, ID,(KARAC(J), J = LRBD,LUPBD)
  303 FORMAT (A3,3X,74A1)
      IF(ID-3HRPC)3,4,3
    3 PRINT 304
  304 FORMAT (17H  RPC IDENT ERROR)
 1090 STOP 2020
    4 LRBD = LRBD+74
      LUPBD = LUPBD+74
    2 CONTINUE
C     THIS SECTION COUNTS NUMBER OF STAGES AND TOTAL NUMBER OF CHARACTER
      KCOUNT = 0
      NSTAGE = 0
      DO 6 I = 1,370
      KCOUNT = KCOUNT+1
      IF(KARAC(I)-1H+)7,8,7
    8 NSTAGE = NSTAGE +1
      GO TO 6
    7 IF(KARAC(I)-1H.)6,9,6
    9 NSTAGE = NSTAGE+1
      GO TO 10
    6 CONTINUE
      PRINT 305
  305 FORMAT(37H RANDOMIZATION PATTERN NOT END WITH .)
 1100 STOP 1100
C     PRESTORE FIRST STAGE DECODING
   10 DO 12 I = 1,NSTAGE
      NSOURCE(I) = 0
      KSRCSZ(I) = 0
      KCELSZ(I) = 0
      NCELLS(I)=0
      KCLFLG(I) = 0
   12 CONTINUE
C     DECODE STAGE (I)
      I = 0
      NLETTR = 0
      LRBD = 1
   13 LRBD = LRBD+NLETTR
      I = I+1
      IF(LRBD-KCOUNT) 14,15,15
   15 PRINT 310
  310 FORMAT(32H TRIED TO DECODE TOO MANY LEVELS)
 1110 STOP 1110
C     FIRST OP CODE MUST BE $
   14 CALL SCAN
      IF(LOPCODE-1H$)17,18,17
   17 PRINT 306
  306 FORMAT(23H  FIRST OP CODE NOT A $)
 1120 STOP 1120
   18 IF(INDCTR-7)20,21,20
C     SET FLAG TO INDICATE CLUSTERING IN STAGE (I)
   21 KCLFLG(I) = 7
      KCELSZ(I) = NLPR
      GO TO 22
   20 NTLPR = NLPR
C     DECODE FROM FIRST TO SECOND OP CODE
   22 LRBD = LRBD+NLETTR
      CALL SCAN
      IF(INDCTR-7) 23,24,23
   24 PRINT 307
  307 FORMAT (45H ILLEGAL SPECIFICATION OF SET AFTER $ OP CODE)
 1130 STOP 1130
   23 IF(KCLFLG(I)) 27,26,27
   26 IF(NTLPR-NLPR) 28,29,28
   28 PRINT 308
  308 FORMAT (30H LEFT AND RIGHT OF $ NOT EQUAL)
 1140 STOP 1140
   29 KCELSZ(I) = NLPR
      KSRCSZ(I) = NLPR
      GO TO 31
   27 NCELLS(I) = NLPR
   31 IF(LOPCODE-1H*)32,33,32
   32 IF(LOPCODE-1H+)34,35,34
   34 IF(LOPCODE-1H.)36,35,36
   36 PRINT 309
  309 FORMAT (16H ILLEGAL OP CODE)
 1150 STOP 1150
C     DECODE SECOND OP CODE WHEN IT ENDS STAGE
   35 NSOURCE(I) = 1
      KSRCSZ(I)=LCMAX
      GO TO 39
   33 LRBD = LRBD+NLETTR
      CALL SCAN
      IF(INDCTR) 24,38,24
   38 NSOURCE(I) = NLPR
      KSRCSZ(I)=LCMAX/NSOURCE(I)
   39 IF(KCLFLG(I)) 40,41,40
   41 NCELLS(I) = NSOURCE(I)
   40 IF(LOPCODE-1H.)42,50,42
   42 IF(LOPCODE-1H+)36,13,36
   50 RETURN
      END
C         SCAN
      SUBROUTINE SCAN
C     INPUT IS RELATIVE POSITION IN LIST TO START SCAN,OUTPUT IS THE
C     PRODUCT OF SUBSCRIPT LEVELS, THE OP CODE,GROUP INDICATOR, AND
C     THE NUMBER OF CHARACTERS SCANNED.
C          LOPCODE         1      SCAN       LAST OP CODE
C          INDCTR          1      SCAN       GROUP INDICATOR
C          NLETTR          1      SCAN       NUMBER OF LETTERS SCANNED
C          LETTRS(8)       1      SCAN       BCD ARRAY OF SUBSCRIPT LETTERS
C          NLPR            1      SCAN       PRODUCT OF LETTERS BETWEEN OP CC
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
      LETTRS(1) = (1HI)
      LETTRS(2) = (1HJ)
      LETTRS(3) = (1HK)
      LETTRS(4) =(1HL)
      LETTRS(5) = (1HM)
      LETTRS(6) = (1HN)
      LETTRS(7) = (1HO)
      NLETTR = 0
      LETTRS(8) = (1HP)
      NLPR = 1
      INDCTR = 0
      DO 1 I = 1,20
      J = LRBD+I-1
      NLETTR = NLETTR +1
      IF(KARAC(J)-1H() 2,3,2
    3 INDCTR = 7
      GO TO 1
    2 IF(KARAC(J)-1H))5,6,5
    6 IF(INDCTR-7)7,1,7
    7 PRINT 307
  307 FORMAT (24H  CLOSING) WITHOUT A (   )
   19 STOP 2030
    5 IF(KARAC(J)-1H$)8,9,8
    8 IF(KARAC(J)-1H*)10,9,10
   10 IF(KARAC(J)-1H+)11,9,11
   11 IF(KARAC(J)-1H.)12,9,12
C     CHARACTER WAS NOT AN OP CODE CHECK FOR SUBSCRIPT LETTER
   12 DO 14 K = 1,8
      IF(KARAC(J)-LETTRS(K))14,15,14
   15 NLPR = NLPR*NL(K+1)
      GO TO 1
   14 CONTINUE
      IF(KARAC(J) -1H ) 16,1,16
   16 PRINT 17,KARAC(J)
   17 FORMAT(15H ILLEGAL LETTER,3X,A1)
 1160 STOP 1160
    1 CONTINUE
      PRINT 308
  308 FORMAT(42H DID NOT FIND OP CODE WITHIN 20 CHARACTERS)
 1170 STOP 1170
    9 LOPCODE = KARAC(J)
      RETURN
      END
C        ANOVAB
      SUBROUTINE ANOVAB
C     READS IN DATA AND POWER TERMS
C         FMT                     ANOVAB     DATA,POWER CARD FORMAT
C         POWER(4096)     1       ANOVAB     TREATMENT EFFECT TERMS
C         DATA(4096)      1       ANOVAB     ORIGINAL PROBLEM DATA
C         KSEQ                    ANOVAB     MONOTONIC SEQUENCE NUMBER
C         LKSEQ                   ANOVAB     SEQUENCE NUMBER PREVIOUS CARD
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
      I=(NCW-1)/8+1
      READ INPUT TAPE 2,
     1     18,FMT
   18 FORMAT(8X,9A8)
      LKSEQ = 0
      DO 20 I = 1,LCMAX
      READ INPUT TAPE 2,
     1     FMT,ID,KSEQ,DATA(I)
      IF(ID-3HDTA) 21,22,21
   21 PRINT 24
   24 FORMAT (22H DATA CARD IDENT ERROR)
 1180 STOP 2040
   22 IF(KSEQ-LKSEQ) 25,25,26
   25 PRINT 27
   27 FORMAT (25H DATA CARD SEQUENCE ERROR)
 1190 STOP 2050
   26 LKSEQ = KSEQ
   20 CONTINUE
      IF(IP) 39,29,39
   29 GO TO 40
   39 LKSEQ = 0
      DO 30 I = 1,LCMAX
      READ INPUT TAPE 2,
     1     FMT,ID,KSEQ,POWER(I)
      IF(ID-3HPOW) 31,32,31
   31 PRINT 34
   34 FORMAT (25H TREAT EFFECT IDENT ERROR)
 1200 STOP 1200
   32 IF(KSEQ-LKSEQ) 35,35,36
   35 PRINT 37
   37 FORMAT (33H TREAT EFFECT CARD SEQUENCE ERROR)
 1210 STOP 1210
   36 LKSEQ = KSEQ
   30 CONTINUE
   40 RETURN
      END
C         ANOVAC
      SUBROUTINE ANOVAC
C     COMPUTES THE ANOVA TABLE
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
      DO 14 J = 1,I1
   14 SUMS(J) = 0.0
      DO 15 J = 1,NCW
   15 LBA(J)=KLBA(J)
      LC=1
      DO 22 I3= 1,LCMAX
      I=LC
      DO 24 J=1,NF
      L=NF+1-J
      IF(XMODF(I,NL(L))) 24,24,26
   24 I=I/NL(L)
   26 DO 28 M=1,NCW
      I=LBA(M)
      SUMS(I) = SUMS(I) + FRATIO(I3,1)
      LBA(M)=I+INC(L,M)
      IF(L-LSQ(M)) 30,30,28
   30 J=LBA(M)
      K=LBASS+(M-1)
      DO 29 N=J,I
      N1=K+XMODF(N-J,1)
      SUMS(N1)=SUMS(N1)+SUMS(N)**2
      IF(L-1) 29,29,32
   32 SUMS(N)=0.0
   29 CONTINUF
   28 CONTINUE
      IF(LCMAX-LC) 31,31,22
   22 LC=LC+1
   31 DO 34 M=1,NCW
      IF ((1).AND.CWD(M)) 36,34
   36 I=LBA(M)
      J=LBA(M)-INC(1,M)
      DO 38 K=I,J
   38 SUMS(K)=SUMS(K)/FLOATF(NDF(M))
   34 CONTINUE
      DO 40 M=1,NCW
      I=LBASS+(M-1)
      J=I
      DO 40 K=I,J
   40 SUMS(K)=SUMS(K)/FLOATF(NDF(M))
      DO 44 M=1,NCW
   44 KDF(M)=LCMAX/(NDF(M))
      DO 46 M=1,NCW
      IF(M-1) 46,46,48
   48 I=M-1
      DO 50 J=1,I
      TMSK=CWD(J).AND..NOT.CWD(M)
      IF(TMSK)50,52,50
   52 TMSK=CWD(M).AND..NOT.CWD(J)
      IF(TMSK) 53,50,53
   53 IF(KOUNT)81,82,81
   82 KDF(M)=KDF(M)-KDF(J)
      DF(M)=FLOATF(KDF(M))
      DF(1)=FLOATF(KDF(1))
   81 L=(M-1)+LBASS
      N=(J-1)+LBASS
   56 SUMS(L)=SUMS(L)-SUMS(N)
   50 CONTINUE
   46 CONTINUE
```

```
      DO 72 M=1,NCW
      I=LBASS+(M-1)
      TMSQ(M)=SUMS(I)/DF(M)
   72 CONTINUE
C     PROGRAM COMPUTES MEAN SQUARE ERROR USING SUMS SPECIFIED BY
C     SUBSCRIPTS PROVIDED BY ETSET
C        ERR                     ERCALC    SUMS OF SQUARES IN ERROR TERM
      DO 1 K = 1,NET
      ERR = 0.0
      EDF(K) = 0.0
      N=IDXE(K,1)+1
      DO 2 J = 2,N
      I = IDXE (K,J)
      L=(I-1)+LBASS
      ERR = ERR+SUMS(L)
    2 EDF(K) = EDF(K) +DF(I)
      EMSQ(K) = ERR/EDF(K)
    1 CONTINUE
C     PROGRAM TO COMPUTE THE FRATIOS AS SPECIFIED BY THE SUBSCRIPTS
C     DETERMINED BY THE FRSET PROGRAM
      DO 11 I = 1,NFR
      J = IFX(I)
      K = KEX(I)
      F(I) = TMSQ (J)/EMSQ(K)
      DFN (I) = DF(J)
      DFD(I) = EDF(K)
   11 CONTINUE
      IF(KOUNT) 101,101,100
  101 PRINT 103
  103 FORMAT(40X,41H ANALYSIS OF VARIANCE TABLE FOR BASE DATA)
      PRINT 104
  104 FORMAT(40X,41H   SOURCE        DF          MS           F      )
      DO 105 I=1,NFR
      J=IFX(I)
      DFN(I)=DF(J)
  105 PRINT 106,KCCW(J),DFN(I),TMSQ(J),F(I)
  106 FORMAT(40X,A8,2X,3X,F4.0,3X,2F10.4)
      DO 107 I=1,NET
  107 PRINT 108,I,EDF(I),EMSQ(I)
  108 FORMAT(40X,8HERROR NO,I2,3X,F4.0,3X,F10.4)
  100 DO 201 I=1,NFR
      FSUM(I)=FSUM(I)+F(I)
      DO 201 J=1,NFR
  201 FCP(I,J)=FCP(I,J)+F(I)*F(J)
      CALL POST
   13 RETURN
C     SPECIFICATIONS, INPUT IS NUMBER OF SOURCES, SOURCE SIZE, NUMBER OF
      END
C       PERMUTE
      SUBROUTINE PERMUTE
C     THIS PROGRAM PERFORMS THE RANDOMIZATION SPECIFIED BY THE RPC
C     CELLS, CELL SIZE, CLUSTERING FLAG FOR ALL STAGES
C        IRNB                     PERMUTE   INTEGER RANDOM NUMBER
C        SOURCE (4096)   1        PERMUTE   SOURCE DATA FOR RANDOMIZING FRO
C        CELL (4096)     1        PERMUTE   ARRAY INTO WHICH DATA PERMUTED
C        FORBID          1        PERMUTE   FORBIDDEN NUMBER
C        NORG                     PERMUTE   SUBSCRIPT FOR ORIGIN OF SOURCE
C        NCRC                     PERMUTE   NUMBER OF SOURCES FOR DO LOOP
C        KSET                     PERMUTE   BLOCK TRANSFER SIZE = CELL SIZE
C        LORG                     PERMUTE   SUBSCRIPT FOR ORIGIN RANDOM CELL.
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C         LBEGN                 PERMUTE    SUBSCRIPT FOR CELL ORIGIN
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4096),POWER(1000),CWD(100),FMT(9),FRATIO(4100,2),
     2DATA(4096),NAME(4),DF(100),TMSQ(100),F(100),KARD(10),KCCW(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KARAC(370),NSOURCE(20),KSRCSZ(20),
     5NCELLS(20),KCELSZ(20),KCLFLG(20),LETTRS(8),KLBA(100),KDF(100)
     6FPP(53),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     7FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION CUMEMSQ(100)
      DIMENSION FTP(11,53,2)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,POWER,FRATIO,CWD,FMT,DATA,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,KARAC,NSTAGE,NSOURCE,KSRCSZ,NCELLS,KCELSZ,KCLFLG,
     4LOPCODE,INDCTR,NLETTR,LETTRS,KCOUNT,NLPR,LRBD,NAME,KLBA,KDF,
     5FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     6KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON CUMEMSQ,NVOID
      COMMON FTP
      COMMON FCORR,FCP,FSUM
      FORBID = -99999.9999
      DO 2 I = 1,NSTAGE
      IF(KCLFLG(I)-7)3,4,3
C     CLUSTERING FLAG NOT SET
    3 NORG = 0
      LBEGN = 0
      NSRC = NSOURCE(I)
C     SOURCES WITHIN STAGE LOOP
      DO 5 J = 1,NSRC
C     OBSERVATIONS WITHIN SOURCE
      KSRK = KSRCSZ(I)
      DO 6 K = 1, KSRK
C     ASSUMED IRAN YIELDS NUMBERS 1-N
    7 IRNBX = KSRCSZ(I)
      IRNB = IRAN(IRNBX)
      L = NORG+IRNB
      IF(FRATIO(L,1)-FORBID) 8,7,8
C     SELECT SOURCE AT RANDOM, FILL CELL IN ORDER
    8 N = NORG+K
      FRATIO(N,2) =FRATIO(L,1)
      FRATIO(L,1) =FORBID
    6 CONTINUE
C     ADVANCE TO ORIGIN NEXT SOURCE WITHIN STAGE
      NORG = NORG+KSRCSZ(I)
    5 CONTINUE
      GO TO 15
C     CLUSTERING FLAG SET
    4 NSRC = NSOURCE(I)
      KSET = KCELSZ(I)
      NCEL = NCELLS(I)
      NORG = 1
      LBEGN = 0
C     SOURCES WITHIN STAGE LOOP
      DO 10 J = 1,NSRC
C     CELLS WITHIN SOURCES LOOP
      DO 11 K = 1,NCEL
   12 IRNBX = NCELLS(I)
      IRNB = IRAN(IRNBX) -1
      LORG = NORG+IRNB*KCELSZ(I)
      IF(FRATIO(LORG,1)-FORBID) 13,12,13
C     LOOP TO TRANSMIT A RANDOM SET OF OBS FROM SOURCE TO CELL
   13 DO 14 L = 1,KSET
      M = LORG+L -1
      N = LBEGN +L
      FRATIO(N,2) =FRATIO(M,1)
      FRATIO(M,1) =FORBID
   14 CONTINUE
      LBEGN = LBEGN+KCELSZ(I)
   11 CONTINUE
      NORG = NORG+KSRCSZ(I)
   10 CONTINUE
   15 DO 16 J = 1,LCMAX
   16 FRATIO(J,1) =FRATIO(J,2)
      IF(IX)19,2,19
   19 IF(KOUNT-10)18,18,2
   18 IF(KOUNT-1)312,311,312
  311 PRINT 310
  310 FORMAT(37H1THE FIRST TEN RANDOMIZATION PATTERNS)
  312 PRINT 308,KOUNT
  308 FORMAT(24H RANDOMIZATION PATTERN   ,I6)
      PRINT 309,(FRATIO(J,1),J=1,LCMAX)
  309 FORMAT(10F11.4)
    2 CONTINUE
   17 RETURN
      END
          FINIS
-CODAP1,L,E.
          IDENT       OCW
          ENTRY       OCW
          BLOCK
          COMMON      INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100)
          COMMON      SUMS(4096),POWER(1000),FRATIO(4100,2)
          COMMON      CWD(100),FMT(9),DATA(4096),NF(1),LCMAX(1),LBASS(1)
          COMMON      I1(1),LC(1),DF(100),TMSQ(100),F(100),KOUNT(1),NCW(1)
          COMMON      IDX(3),IDXE(100,20),IDC(1),EMSQ(100),EDF(100),
          COMMON      DFN(100),DFD(100),KCCW(100),KEX(100),IFX(100),ID(1)
          COMMON      NET(1),NFR(1),NRPC(1),IP(1),NSAMP(1),IR(1),IPOST(1)
          COMMON      IX(1),IY(1),IZ(1),KRAND(1),KARAC(370),NSTAGE(1)
          COMMON      NSOURCE(20),KSRCSZ(20),NCELLS(20),KCELSZ(20)
          COMMON      KCLFLG(20),LOPCODE(1),INDCTR(1),NLETTR(1),LETTRS(8)
          COMMON      KCOUNT(1),NLPR(1),LRBD(1),NAME(4)
          COMMON      KLBA(100),KDF(100),FPP(53),IDXFR(100),XLB(50)
          COMMON      XUB(50),FCOUNT(50),FOBS(20),FTHEORY(20),FX(100)
          COMMON      KARD(10),KEOP(7),KECW(7)
          COMMON      KFTH(20),FKFTH(20),PCFTH(20)
          COMMON      CUMEMSQ(100),NVOID(1)
          COMMON      FTP(11,53,2)
          COMMON      FCORR(10,10),FCP(10,10),FSUM(10)
OCW       SLJ         **
          SIU     1   IDX                           SAVE
          SIL     2   IDX                           INDEX
          SIU     3   IDX+1                         REGISTERS
          SIL     4   IDX+1
          SIU     5   IDX+2
          SIL     6   IDX+2
          LDA         NCW                           NCW-1 FOR INDEX
          INA         77776B
          SAU         DLTA                          STORE INDEX
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         ENI     2  INC+1                   ENTER INC ORIGIN
         ENI     3  0                       CLEAR B3
ALPHA    SIU     2  BETA                    SET INC WORD ORIGIN IN ADD
         ENA        0
         ENI     1  0                       CLEAR B1
BETA     ADD     1  *                       ADD CHARACTER TO A
         ALS        3                       LEFT SHIFT 3
         ISK     1  7                       DONE ALL OF A WORD
         SLJ        BETA                    NO
         ALS        3                       YES LEFT JUSTIFY TO 10
         STA     3  CWD                     POSITION
DLTA     ISK     3  **                      DONE ALL WORDS
         SLJ        GAM                     NO
         LIU     1  IDX                     YES RESTORE
         LIL     2  IDX                     INDEX
         LIU     3  IDX+1                   REGISTERS
         LIL     4  IDX+1
         LIU     5  IDX+2
         LIL     6  IDX+2
         SLJ        OCW                     TO EXIT
GAM      INI     2  13B                     ADVANCE WORD
         SLJ        ALPHA                   ORIGIN
         END
         IDENT      IRAN
         ENTRY      IRAN
         BLOCK
         COMMON     INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100)
         COMMON     SUMS(4096),POWER(1000),FRATIO(4100,2)
         COMMON     CWD(100),FMT(9),DATA(4096),NF(1),LCMAX(1),LBASS(1)
         COMMON     I1(1),LC(1),DF(100),TMSQ(100),F(100),KOUNT(1),NCW(1)
         COMMON     IDX(3),IDXE(100,20),IDC(1),EMSQ(100),EDF(100),
         COMMON     DFN(100),DFD(100),KCCW(100),KEX(100),IFX(100),ID(1)
         COMMON     NET(1),NFR(1),NRPC(1),IP(1),NSAMP(1),IR(1),IPOST(1)
         COMMON     IX(1),IY(1),IZ(1),KRAND(1),KARAC(370),NSTAGE(1)
         COMMON     NSOURCE(20),KSRCSZ(20),NCELLS(20),KCELSZ(20)
         COMMON     KCLFLG(20),LOPCODE(1),INDCTR(1),NLETTR(1),LETTRS(8)
         COMMON     KCOUNT(1),NLPR(1),LRBD(1),NAME(4)
         COMMON     KLBA(100),KDF(100),FPP(53),IDXFR(100),XLB(50)
         COMMON     XUB(50),FCOUNT(50),FOBS(20),FTHEORY(20),FX(100)
         COMMON     KARD(10),KEOP(7),KECW(7)
         COMMON     KFTH(20),FKFTH(20),PCFTH(20)
         COMMON     CUMEMSQ(100),NVOID(1)
         COMMON     FTP(11,53,2)
         COMMON     FCORR(10,10),FCP(10,10),FSUM(10)
IRAN     SLJ        **
         SIL     1  EXIT                    ENTRY
         LIU     1  IRAN
         LDA     1  0
         LRS        24
         INI     1  1
         SIU     1  EXIT+1
         SAU        TAG
         LDA        =D1220703125
         MUF        KRAND
         ENA        0
         LRS        1
         STQ        KRAND
TAG      LDA        *
         MUF        KRAND
EXIT     INA        1
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```
FNI     1  **
SLJ        **
FND
END
FINIS
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PROGRAM REPMEAS

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      PROGRAM REPMEAS
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(4000),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),            C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
  997 READ 1, ID, NET, NFR, NRPC, IP,NSAMP, IR, IPOST, IX, IY, IZ,KRAND
      PRINT 207,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,IZ,KRAND
  207 FORMAT(1H1,A3,I2,2I3,I1,I4,5I1,I10)
    1 FORMAT (A3, I2, 2I3, I1, I4,5I1, I10)
      NSAMP=NSAMP-1
      IF(ID-3HPDC)2,3,2
    3 KOUNT = 0
      DO 224 I=1,NET
  224 CUMEMSQ(I)=0.0
      DO 225 I=1,NFR
      FSUM(I)=0.0
      DO 225 J=1,NFR
  225 FCP(I,J)=0.0
      DO 606 I=1,14
  606 EP(I)=0.0
      CALL ANOVAA
      CALL ETSET
      CALL FRSET
      CALL FACTOR
      WASTE=RANDEV(KRAND)
      DO 29 I=1,NBET
      DO 29 K=1,NWTH
      CSUM(I,K)=0.0
      DO 28 L=1,NWTH
   28 CPROD(I,K,L)=0.0
      FTP(11,17,2)=0.0
      FTP(11,18,2)=0.0
   29 CONTINUE
   60 IF(NOP) 31,30,31
   30 DO 32 I=2,NBET
      DO 32 J=1,NWTH
      DO 32 K=1,NWTH
   32 AFACT(I,J,K)=AFACT(1,J,K)
   31 DO 600 I=1,NWTH
      CS(I)=0.0
      DO 600 K=1,NWTH
  600 CP(I,K)=0.0
      NDC=0
      DO 37 I=1,NBET
      DO 48 J=1,NWTH
      DO 49 K=1,NWTH
   49 CFACT(J,K)=AFACT(I,J,K)
   48 SVECT(J)=VECTOR(I,J)
      DO 36 J=1,NSUBJ
      CALL DGEN
  403 DO 34 K=1,NWTH
      M=K+NDC
      FRATIO(M)=Y(K)
   55 CSUM(I,K)=CSUM(I,K)+Y(K)
      CS(K)=CS(K)+Y(K)
      DO 35 L=1,NWTH
      CP(K,L)=CP(K,L)+Y(K)*Y(L)
   35 CPROD(I,K,L)=CPROD(I,K,L)+Y(K)*Y(L)
   34 CONTINUE
      NDC=NDC+NWTH
   36 CONTINUE
   37 CONTINUE
      FNS=FLOATF(NSUBJ*NBET)
      DO 602 K=1,NWTH
      DO 602 L=1,NWTH
  602 CV(K,L)=(FNS*CP(K,L)-CS(K)*CS(L))/(FNS*(FNS-1.0))
      VZ = 0.0
      VW = 0.0
      VY = 0.0
      VX = 0.0
      VT = 0.0
      FNWTH = NWTH
      DO 500 I = 1,NWTH
      DO 501 J = 1,NWTH
      VX = CV(I,J)+VX
      VY = CV(I,J)**2+VY
  501 CONTINUE
      VZ = 2.0*(VX**2)/FNWTH+VZ
      VT = CV(I,I)+VT
      VW = VW+VX
      VX = 0.0
  500 CONTINUE
      EPS = ((VT-VW/FNWTH)**2)/((FNWTH-1.0)*(VY-VZ+ (VW/FNWTH)**2))
      FTP(11,17,2)=FTP(11,17,2)+EPS
      FTP(11,18,2)=FTP(11,18,2)+EPS**2
      EPS=EPS-0.30
      IF(EPS)605,605,607
  607 DO 603 J=1,13
      FJ=J
      IF(EPS-0.05*FJ) 604,604,603
  604 EP(J)=EP(J)+1.0
      GO TO 605
  603 CONTINUE
      EP(14)=EP(14)+1.0
  605 IF(KOUNT) 150,150,999
  999 IF(KOUNT-999) 9,150,150
  150 FKOUNT=FLOATF(KOUNT+1)
      FNSUBJ=NSUBJ
      DO 39 I=1,NBET
      DO 38 K=1,NWTH
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      CMEAN(I,K)=CSUM(I,K)/(FNSUBJ*FKOUNT)
      DO 38 L=1,NWTH
   38 COV(I,K,L)=(FKOUNT*FNSUBJ*CPROD(I,K,L)-CSUM(I,K)*CSUM(I,L))/
     1((FKOUNT*FNSUBJ)*(FKOUNT*FNSUBJ-1.0))
   39 CONTINUE
      PRINT 40,KOUNT
   40 FORMAT(62H0POOLED SAMPLE MEAN VECTORS AND COVARIANCE MATRICES FOR
     1KOUNT=,I5)
      DO 41 I=1,NBET
      PRINT 42,I,(CMEAN(I,K),K=1,NWTH)
   42 FORMAT(//2X,13H0GROUP NUMBER,I4,10F10.4)
      PRINT 75
   75 FORMAT(18H0COVARIANCE MATRIX)
      DO 57 K=1,NWTH
   57 PRINT 58,(COV(I,K,L),L=1,NWTH)
   58 FORMAT(10F10.4)
   41 CONTINUE
    9 CALL ANOVAC
C     THIS BEGINNING OF SEGMENT 2
   12 KOUNT = KOUNT + 1
      IF(NSAMP-KOUNT)16,60,60
   16 FNUM=FLOATF(NSAMP+1)
      DO 7005 K=1,NET
 7005 CUMEMSQ(K)=CUMEMSQ(K)/FNUM
      PRINT 7006
 7006 FORMAT(27H0AVERAGE ERROR MEAN SQUARES)
      PRINT 7007,(CUMEMSQ(K),K=1,NET)
 7007 FORMAT(10F11.4)
      DO 7008 I=1,NFR
      DO 7008 J=1,NFR
 7008 FCORR(I,J)=(FNUM*FCP(I,J)-FSUM(I)*FSUM(J))/SQRTF((FNUM*FCP(I,I)
     1-FSUM(I)**2)*(FNUM*FCP(J,J)-FSUM(J)**2))
      PRINT 7009
 7009 FORMAT(16H0F CORREL MATRIX)
      DO 7010 I=1,NFR
 7010 PRINT 7011,(FCORR(I,J),J=1,NFR)
 7011 FORMAT(10F10.4)
      GO TO 997
    2 END
      SUBROUTINE ANOVAA
C     JENNRICKS ANOVA HAS BEEN BROKEN UP INTO FOUR SUBROUTINES THE FIRST
C     (ANOVAA) READS IN THE CODE WORDS AND SETS UP THE ARRAYS OF DIGITS
C     AND OCTAL WORDS , THE SECOND(ANOVAB) USES THE CODE WORDS ETC TO
C     SET UP THE SUMS OF SQUARES CALCULATIONS, THE THIRD(ANOVAC) READS
C     THE DATA AND POWER TERNS IF ANY, THE FOURTH(ANOVAD) COMPUTES THE
C     COMPLETE ANOVA TABLE USING THE ERROR TERM AND F RATIOS SPECIFIED
C     ON THE INPUT CARDS
C        KARD(10)                ANOVAY    BCD CARD IMAGE ARRAY
C        KCRD                    ANOVAY    COMPUTED NUMBER OF CARDS
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),     C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
C     READS IN CONTROL CARD,CONTROL WORDS AND SETS UP THE ARRAYS
C     INC,CWD,KCCW FOR USE BY THE ANOVA SUBROUTINES
      READ 1,ID,NF,NDV,NCW,(NL(I),I = 2,9), NAME
      PRINT 1,ID,NF,NDV,NCW,(NL(I),I=2,9),NAME
    1 FORMAT(A3,I1,10I4,4A8)
      IF(ID-3HAVA) 13,12,13
   13 PRINT 14
   14 FORMAT(28H  IMPROPER IDENT ON AVA CARD)
 1020 STOP 1020
C     THE ACW CARDS ARE READ FROM TAPE 2 INTO KCCW THEN WRITTEN
C     ON THE USERS OUTPUT TAPE AS A SCRATCH AND RE-READ INTO INC AS
C     THE SAME INFORMATION MUST BE READ UNDER TWO FORMATS.
   12 READ 4,(KCCW(I),I=1,NCW)
    4 FORMAT (4X,9A8,4X)
C     CLEAR INTEGER CODE WORD ARRAY
      DO 5 I = 1,9
      DO 5 J = 1,100
    5 INC (I,J) = 0
      READ 6,((INC(L,M),L=2,9),M=1,NCW)
    6 FORMAT(4X,72A1,4X)
      PRINT 4,(KCCW(I),I=1,NCW)
      DO 9 J = 1,NCW
      DO 9 I = 2,9
      IF(INC(I,J)-1H0) 8,7,8
    7 INC (I,J) = 0
      GO TO 9
    8 INC (I,J) = 1
    9 CONTINUE
      CALL OCW
      PRINT 11, (CWD(I), I = 1,NCW)
   11 FORMAT(O16)
      NL(1)=1
    2 NF=NF+1
      LCMAX=1
      DO 35 L=1,NF
   35 LCMAX=LCMAX*NL(L)
      DO36 M=1,NCW
      I=1
      DO37 L=1,NF
   37 I=I*NL(L)**INC(L,M)
      NDF(M)=LCMAX/I
      IF(INC(1,M)) 38,38,39
   38 DO 10 L=2,NF
      IF(INC(L,M)) 41,41,10
   10 INC(L,M)=0
   21 LSQ(M)=NF
      GO TO 19
   41 LSQ(M)=L-1
      GO TO 19
   39 LSQ(M)=1
      INC(1,M)=0
   19 DO36 L=1,NF
      I=1-INC(L,M)
      DO 52 J=L,NF
   52 I=I*NL(J)**INC(J,M)
   36 INC(L,M)=1-I
      LBA(1)=1
      DO 53 M=2,NCW
   53 LBA(M)=LBA(M-1)+1-INC(1,M-1)
      DO 54 K=1,NCW
   54 KLBA(K)=LBA(K)
      LBASS=LBA(NCW)-INC(1,NCW)+1
      I1=LBASS+NCW
      IF(4096-I1) 25,27,27
   25 PRINT 33
   33 FORMAT(//39HTHIS DESIGN IS TOO LARGE FOR THE MEMORY)
 1030 STOP 1030
   27 RETURN
      END
      MACHINE OCW
      LOC(ZQQ=0)
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),              C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
2ETR    SLJ (*         ), ENI (0         ).ENTRY
        SIU1(IDX       ), SIL2(IDX       ).SAVE
        SIU3(IDX+1     ), SIL4(IDX+1     ).INDEX
        SIU5(IDX+2     ), SIL6(IDX+2     ).REGISTERS
        LDA (NCW       ), SUB (1          ). NCW-1 FOR INDEX
        SAU (2DLTA     ), ENI (0       ). STORE INDEX
        ENI2(INC+2     ), ENI3(0         ).ENTER INC ORIGIN, CLEAR B3
2ALPH   SIU2(2BETA     ), ENA (0         ).SET INC WORD ORIGIN IN ADD
        ENI1(0         ), ENI (0         ).CLEAR B1 CHARACTER COUNT
2BETA   ADD1(*         ), ALS (3         ).ADD CHARACTER TO A,LEFT SHIFT3
        ISK1(7         ), SLJ (2BETA     ).DONE ALL OF A WORD NO
        ALS (3B       ), STA3(CWD+1      ). YES LEFT JUSTIFY TO 10 POSITI
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2DLTA   ISK3(*            ), SLJ (2GAM          ).DONE ALL WORDS NO
        LIU1(IDX          ), LIL2(IDX           ).YES RESTORE
        LIU3(IDX+1        ), LIL4(IDX+1         ).INDEX
        LIU5(IDX+2        ), LIL6(IDX+2         ).REGISTERS
        SLJ (2ETR         ), ENI (0             ).TO EXIT
2GAM    INI2(13B          ), SLJ (2ALPH         ).ADVANCE WORD ORIGIN
      END
      SUBROUTINE ETSET
C     A PROGRAM TO ESTABLISH THE ARRAY OF SUBSCRIPTS TO BE USED IN THE
C     ERROR TERM CALCULATIONS. ASSUMES ARRAY KCCW IN MEMORY
C         KNTE                  ETSET     COUNT OF ERROR TERMS READ IN
C         NER                   ETSET     ERRTERM SUBSCRIPT ON INPUT CARD.
C         KECW                  ETSET     ERROR TERM CODE WORD
C         KEOP                  ETSET     OP CODE IN ERROR TERM SPECIFICAT
C         KER                   ETSET     ERROR TERM SUBSCRIPT
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),                C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NER,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
    1 KNTE = 0
      K=1
    3 IF(KNTE-NET) 7,26,7
    7 READ 9,ID,IDC,NER,(KECW(I),KEOP(I),I = 1,7)
    9 FORMAT(A3,2X,I1,2X,I2,2X,7(A8,A1))
      IF(ID-3HETC) 10,11,10
   10 PRINT 12
   12 FORMAT (32H  READ IMPROPER ERROR TERM IDENT)
 1040 STOP 1040
   11 IF(IDC-1)14,15,14
   14 KNTE = KNTE +1
      KER = NER
C     TABLE LOOK UP OF CODE WORD TO DETERMINE SUBSCRIPT
   15 I = 0
   16 K = K+1
      I = I+1
      DO 17 J=1,NCW
      IF(KECW(I)-KCCW(J))17,19,17
   19 IDXE(KER,K)=J
      PRINT 30, KNTE,KECW(I),KER,K,IDXE(KER,K),KEOP(I)
   30 FORMAT(I5,2X,A8,2X,I5,I5,I5,2X,A1)
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      IF(KEOP(I)-1H+)20,21,20
   21 IF(I-7) 16,7,16
   20 IF(KEOP(I)-1H.)22,23,22
   22 PRINT 24
   24 FORMAT (21H  OP CODE NOT + NOT .)
 1050 STOP 1050
   23 IDXE(KER,1)=K-1
      K=1
      GO TO 3
   17 CONTINUE
      PRINT 28
   28 FORMAT (50H ERROR TERM CODE WORD AND ANOVA CODE WORD NO MATCH)
 1060 STOP 1060
   26 PRINT 27,((IDXE(I,J),J=1,20),I=1,NET)
   27 FORMAT(20I5)
      RETURN
      END
      SUBROUTINE FRSET
C     PROGRAM TO SET UP SUBSCRIPTS OF MEAN SQUARES AND ERROR TERMS
C     USED TO COMPUTE F RATIOS
C           KFX             FRSET         SUBSCRIPT OF F RATIO
C           KNCW            FRSET         UPR LIM ON CWD WRDS SRCHED
C           KFCW            FRSET         MEAN SQUARE CODE WORD
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),           C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
    1 DO 12 I = 1,NFR
      READ 2,ID,KFX,KFCW,KEX(KFX)
    2 FORMAT (A3,5X,I2,2X,A8,3X,I2)
      IF(ID-3HFRC)10,9,10
    9 DO 3 K = 1,NCW
      IF(KFCW-KCCW(K))3,5,3
    5 IFX (KFX) = K
      PRINT 6,IFX(KFX),KEX (KFX)
    6 FORMAT (2I10)
      GO TO 12
    3 CONTINUE
    7 PRINT 14
   14 FORMAT (48H  F RATIO CODE WORD NOT MATCH ANOVA CONTROL WORD)
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1080 STOP 1080
  12 CONTINUE
     GO TO 13
  10 PRINT 11
  11 FORMAT (31H  IMPROPER FRATIO IDENT ON CARD)
1070 STOP 1070
  13 RETURN
     END
     SUBROUTINE ANOVAC
C    COMPUTES THE ANOVA TABLE
     DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
    1SUMS(4000),KCCW(100),CWD(100),FRATIO(4000),
    2NAME(4),DF(100),TMSQ(100),F(100),
    3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
    4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
    5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
    6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
     DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
    1A(10,10,10),            C(10,10),AFACT(10,10,10),CSUM(10,10),
    2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
     DIMENSION FTP(11,20,6)
     DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
     DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
     COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
    1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
    2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
    3IZ,KRAND,NAME,KLBA,KDF,
    4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
    5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
     COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
    1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
     COMMON FTP
     COMMON CS,CP,CV,EP
     COMMON FCORR,FCP,FSUM,CUMEMSQ
     DO 14 J = 1,I1
  14 SUMS(J) = 0.0
     DO 15 J = 1,NCW
  15 LBA(J)=KLBA(J)
     LC=1
     DO 22 I3= 1,LCMAX
     I=LC
     DO 24 J=1,NF
     L=NF+1-J
     IF(XMODF(I,NL(L))) 24,24,26
  24 I=I/NL(L)
  26 DO 28 M=1,NCW
     I=LBA(M)
     SUMS(I)=SUMS(I)+FRATIO(I3)
     LBA(M)=I+INC(L,M)
     IF(L-LSQ(M)) 30,30,28
  30 J=LBA(M)
     K=LBASS+(M-1)
     DO 29 N=J,I
     N1=K+XMODF(N-J,1)
     SUMS(N1)=SUMS(N1)+SUMS(N)**2
     IF(L-1) 29,29,32
  32 SUMS(N)=0.0
  29 CONTINUE
  28 CONTINUE
     IF(LCMAX-LC) 31,31,22
```

```
   22 LC=LC+1
   31 DO 34 M=1,NCW
B     IF(1*CWD(M)) 34,34,36
   36 I=LBA(M)
      J=LBA(M)-INC(1,M)
      DO 38 K=I,J
   38 SUMS(K)=SUMS(K)/FLOATF(NDF(M))
   34 CONTINUE
      DO 40 M=1,NCW
      I=LBASS+(M-1)
      J=I
      DO 40 K=I,J
   40 SUMS(K)=SUMS(K)/FLOATF(NDF(M))
      DO 44 M=1,NCW
   44 KDF(M)=LCMAX/(NDF(M))
      DO 46 M=1,NCW
      IF(M-1) 46,46,48
   48 I=M-1
      DO 50 J=1,I
B     IF(CWD(J)*(-CWD(M))*(-1)) 52,52,50
B  52 IF(CWD(M)*(-CWD(J))*(-1)) 50,50,53
   53 IF(KOUNT)81,82,81
   82 KDF(M)=KDF(M)-KDF(J)
      DF(M)=FLOATF(KDF(M))
      DF(1)=FLOATF(KDF(1))
   81 L=(M-1)+LBASS
      N=(J-1)+LBASS
   56 SUMS(L)=SUMS(L)-SUMS(N)
   50 CONTINUE
   46 CONTINUE
      DO 72 M=1,NCW
      I=LBASS+(M-1)
      TMSQ(M)=SUMS(I)/DF(M)
   72 CONTINUE
C     PROGRAM COMPUTES MEAN SQUARE ERROR USING SUMS SPECIFIED BY
C     SUBSCRIPTS PROVIDED BY ETSET
C        ERR                  ERCALC     SUMS OF SQUARES IN ERROR TERM
      DO 1 K = 1,NET
      ERR = 0.0
      EDF(K) = 0.0
      N=IDXE(K,1)+1
      DO 2 J = 2,N
      I = IDXE (K,J)
      L=(I-1)+LBASS
      ERR = ERR+SUMS(L)
    2 EDF(K) = EDF(K) +DF(I)
      EMSQ(K) = ERR/EDF(K)
      CUMEMSQ(K)=CUMEMSQ(K)+EMSQ(K)
    1 CONTINUE
C     PROGRAM TO COMPUTE THE FRATIOS AS SPECIFIED BY THE SUBSCRIPTS
C     DETERMINED BY THE FRSET PROGRAM
      DO 11 I = 1,NFR
      J = IFX(I)
      K = KEX(I)
      F(I) = TMSQ (J)/EMSQ(K)
      DFN (I) = DF(J)
      DFD(I) = EDF(K)
   11 CONTINUE
      DO 201 I=1,NFR
      FSUM(I)=FSUM(I)+F(I)
      DO 201 J=1,NFR
  201 FCP(I,J)=FCP(I,J)+F(I)*F(J)
      CALL POST
      IF(KOUNT)4,4,13
    4 PRINT 101
  101 FORMAT(21HOF-RATIOS FOR KOUNT=0)
      DO 102 N=1,NCW
  102 PRINT 103,KOUNT,KCCW(N),TMSQ(N),EMSQ(N),F(N),DF(N),DFN(N),DFD(N),
     1KDF(N),NET
  103 FORMAT(1X,I6,3X,A8,6E13.4,2I10)
   13 RETURN
      END
      SUBROUTINE FACTOR
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),          C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
      READ 99,NBET,NWTH,NSUBJ,NOP
   99 FORMAT(2I2,I3,I1)
      IF(NOP)3,2,3
    2 DO 4 I=1,NBET
    4 READ 5,(VECTOR(I,J),J=1,NWTH)
    5 FORMAT(10F6.2)
      DO 6 J=1,NWTH
    6 READ 7,(A(1,J,K),K=1,NWTH)
    7 FORMAT(20F4.0)
      DO 17 J=1,NWTH
      DO 17 K=1,NWTH
   17 C(J,K)=A(1,J,K)
      CALL FACTORS
      DO 18 J=1,NWTH
      DO 18 K=1,NWTH
   18 AFACT(1,J,K)=CFACT(J,K)
      GO TO 22
    3 DO 11 I=1,NBET
      READ 5,(VECTOR(I,J),J=1,NWTH)
      DO 10 J=1,NWTH
   10 READ 7,(A(I,J,K),K=1,NWTH)
   11 CONTINUE
   16 DO 21 I=1,NBET
```

```
      DO 19 J=1,NWTH
      DO 19 K=1,NWTH
   19 C(J,K)=A(I,J,K)
      CALL FACTORS
      DO 20 J=1,NWTH
      DO 20 K=1,NWTH
   20 AFACT(I,J,K)=CFACT(J,K)
   21 CONTINUE
   22 END
      SUBROUTINE FACTORS
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),            C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
      DO 1  I=1,NWTH
      DO 1  J=1,NWTH
    1 CFACT(I,J)=0.0
      CFACT(1,1)=SQRTF(ABSF(C(1,1)))
      DO 5 I=2,NWTH
    5 CFACT(I,1)=C(I,1)/CFACT(1,1)
      DO 6 I=2,NWTH
      TEMP=0.0
      JA=I-1
      DO 7 J=1,JA
    7 TEMP=TEMP+CFACT(I,J)*CFACT(I,J)
      CFACT(I,I)=SQRTF(ABSF(C(I,I)-TEMP))
      IF(NWTH-I)9,9,8
    8 KA=I+1
      DO 6 K=KA,NWTH
      TEMP=0.0
      DO 10 L=1,JA
   10 TEMP=TEMP+CFACT(I,L)*CFACT(K,L)
    6 CFACT(K,I)=(C(K,I)-TEMP)/CFACT(I,I)
    9 PRINT 13
   13 FORMAT(42H0INPUT COVARIANCE MATRIX AND FACTORIZATION)
      DO 14 I=1,NWTH
   14 PRINT 15,(C(I,J),J=1,NWTH)
   15 FORMAT(10F10.4)
      DO 16 I=1,NWTH
   16 PRINT 15,(CFACT(I,J),J=1,NWTH)
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      PRINT 20
   20 FORMAT(20HOFACTOR MATRIX CHECK)
      DO 17 I=1,NWTH
      DO 18 J=1,NWTH
      CHECK(I,J)=0.0
      DO 18 K=1,NWTH
   18 CHECK(I,J)=CHECK(I,J)+CFACT(I,K)*CFACT(J,K)
   17 PRINT 15,(CHECK(I,L),L=1,NWTH)
      END
      SUBROUTINE DGEN
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(10,40,10),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),            C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
      DO 22 I=1,NWTH
      TEMP1=RANDEV(-1)
   22 X(I)=TEMP1
      DO 201 IA=1,NWTH
      Y(IA)=0.0
      DO 23 J=1,NWTH
   23 Y(IA)=Y(IA)+CFACT(IA,J)*X(J)
  201 Y(IA)=Y(IA)+SVECT(IA)
      END
      SUBROUTINE POST
      DIMENSION INC(11,100),LBA(100),NL(11),LSQ(100),NDF(100),
     1SUMS(4000),KCCW(100),CWD(100),FRATIO(1000,4),
     2NAME(4),DF(100),TMSQ(100),F(100),
     3IDX(3),IDXE(100,20),KECW(7),KEOP(7),EMSQ(100),KEX(100),IFX(100),
     4DFN(100),DFD(100),EDF(100),KLBA(100),KDF(100),
     5FPP(20),IDXFR(100),XLB(50),XUB(50),FCOUNT(50),FOBS(20),
     6FTHEORY(20),FX(100),KFTH(20),FKFTH(20),PCFTH(20)
      DIMENSION X(10),Y(10),CFACT(10,10),SVECT(10),VECTOR(10,10),
     1A(10,10,10),            C(10,10),AFACT(10,10,10),CSUM(10,10),
     2CPROD(10,10,10),CMEAN(10,10),COV(10,10,10),CHECK(10,10)
      DIMENSION FTP(11,20,6)
      DIMENSION CS(10),CP(10,10),CV(10,10),EP(14)
      DIMENSION FCORR(10,10),FCP(10,10),FSUM(10),CUMEMSQ(10)
      COMMON INC,LBA,NL,LSQ,NDF,SUMS,FRATIO,CWD,
     1NF,LCMAX,LBASS,I1,LC,DF,TMSQ,F,KOUNT,NCW,IDX,IDXE,IDC,EMSQ,EDF,
     2DFN,DFD,KCCW,KEX,IFX,ID,NET,NFR,NRPC,IP,NSAMP,IR,IPOST,IX,IY,
     3IZ,KRAND,NAME,KLBA,KDF,
     4FPP,IDXFR,XLB,XUB,FCOUNT,FOBS,FTHEORY,FX,
     5KARD,KEOP,KECW,KFTH,FKFTH,PCFTH
      COMMON X,Y,CFACT,SVECT,VECTOR,A,C,AFACT,CSUM,CPROD,
     1CMEAN,COV,CHECK,NBET,NWTH,NSUBJ,NOP
      COMMON FTP
      COMMON CS,CP,CV,EP
      COMMON FCORR,FCP,FSUM,CUMEMSQ
      IF (KOUNT) 150,150,151
  150 PRINT 499
  499 FORMAT(18H1POST CONTROL CARD)
      READ 1,ID,IP1,IP2,NUMSP,IP3,IP4,(FPP(I),I =13,18),IP5,IP6,
     1(IDXFR(J),J=1,10),IP7
    1 FORMAT(A3,2I1,I4,2I1,6F4.4,I2,I3,11I1)
      PRINT 1,ID,IP1,IP2,NUMSP,IP3,IP4,(FPP(I),I = 13,18),IP5,IP6,
     1(IDXFR(J),J=1,10),IP7
      IF(ID-3HPST)2,3,2
    2 PRINT 4
    4 FORMAT (25H.POST CONTROL IDENT ERROR)
    5 STOP 5
    3 IF(IP3-1) 9,6,9
    6 READ 8,ID,(IDXFR(J), J = 41,100)
    8 FORMAT (A3,7X,60I1)
      PRINT 8,ID, (IDXFR(J) , J = 41,100)
      IF(ID-3HPST) 2,9,2
    9 PRINT 10,ID,NAME,NF,NCW,NFR,NSAMP,KOUNTP,KRAND
      PRINT 201
  201 FORMAT(37H0TAPE 4 HEADER FOLLOWED BY POST INPUT)
      DO 200 N=1,NCW
  200 PRINT 28,KOUNT,KCCW(N),DF(N),TMSQ(N),FX(N),DFN(N),DFD(N)
   10 FORMAT(A3,3X,4A8/6I10)
   28 FORMAT(1X,I6,3X,A8,5E13.4)
      NUMSP=NUMSP-1
      IF(NUMSP-NSAMP)11,11,12
   12 PRINT 13
   13 FORMAT (45H ASKED TO ANALYZE MORE SAMPLES THAN AVAILABLE)
   14 STOP 14
   11 KKK=12+IP4
C     SORT F PER CENT POINTS INTO ASCENDING ORDER
      FPP(1)=.100
      FPP(2)=.200
      FPP(3)=.300
      FPP(4)=.400
      FPP(5)=.500
      FPP(6)=.600
      FPP(7)=.700
      FPP(8)=.800
      FPP(9)=.900
      FPP(10)=.950
      FPP(11)=.975
      FPP(12)=.990
   16 DO 15 I = 1,KKK
      DO 17 J = I,KKK
      IF(FPP(I)-FPP(J))17,17,18
   18 TEMP1 = FPP(I)
      FPP(I) = FPP(J)
      FPP(J) = TEMP1
   17 CONTINUE
   15 CONTINUE
      FTP(11,20,2)=KKK
      FTP(11,19,2)=IP6
C     CONVERT F SPECS FROM 1 AND 0 TO LOCATION SUBSCRIPTS
      IF(IP6-NFR) 19,19,20
   20 PRINT 21
   21 FORMAT (28H ASKED FOR TOO MANY F RATIOS)
   22 STOP 22
   19 J=1
      LFLST=0
      DO 123 I=1,NFR
      IF(IDXFR(I)) 24,23,24
   24 IDXFR(J) = I
      LFLST=LFLST+1
   23 J=J+1
  123 CONTINUE
      DO 156 I=1,10
      DO 156 J=1,20
      FTP(I,J,6)=0.0
      FTP(I,J,4) = 0.0
  156 FTP(I,J,2) =0.0
      IF(IP7) 166,166,564
  166 DO 49 I=1,IP6
      NI=IDXFR(I)
      DO 49 K = 1,KKK
      P = FPP(K)
      FC= 0.0
      M=DFN(NI)
      N = DFD(NI)
      CALL FDISTR(M,N,FC,P)
      FTP(I,K,1)=FC
   49 CONTINUE
  564 IF(NOP) 600,600,601
  601 DO 602 J=1,NWTH
      DO 602 K=1,NWTH
  602 C(J,K)=0.0
      FNBET=NBET
      DO 603 J=1,NWTH
      DO 603 K=1,NWTH
      DO 603 I=1,NBET
  603 C(J,K)=A(I,J,K)/FNBET + C(J,K)
  600 VZ=0.0
      VW = 0.0
      VY = 0.0
      VX = 0.0
      VT = 0.0
      FNWTH = NWTH
      DO 500 I = 1,NWTH
      DO 501 J = 1,NWTH
      VX = C(I,J)+VX
      VY = C(I,J)**2+VY
  501 CONTINUE
      VZ = 2.0*(VX**2)/FNWTH+VZ
      VT = C(I,I)+VT
      VW = VW+VX
      VX = 0.0
  500 CONTINUE
      EPS = ((VT-VW/FNWTH)**2)/((FNWTH-1.0)*(VY-VZ+ (VW/FNWTH)**2))
      IF(IP7) 570,570,151
  570 DO 502 I=2,IP6
      NI = IDXFR(I)
      DO 502 K = 1,KKK
```

```
    P = FPP(K)
    FC = 0.0
    M=XFIXF(DFN(NI)*EPS+0.5)
    N=XFIXF(DFD(NI)*EPS+0.5)
573 CALL FDISTR(M,N,FC,P)
502 FTP(I,K,3) = FC
    DO 511 K=1,KKK
511 FTP(1,K,3)=FTP(1,K,1)
    DO 574 J=1,KKK
    FTP(1,J,5)=FTP(1,J,1)
    FTP(3,J,5)=FTP(1,J,1)
    P=FPP(J)
    NI=IDXFR(2)
    FC=0.0
    M=DFN(NI)
    M=M/(NWTH-1)
    N=DFD(NI)
    N=N/(NWTH-1)
    CALL FDISTR(M,N,FC,P)
574 FTP(2,J,5)=FC
151 KKK=FTP(11,20,2)
    IP6=FTP(11,19,2)
    KA=KKK+1
    DO 34 L=1,IP6
    FTP(11,L,1)=F(L)
 34 CONTINUE
    DO 575 L=1,3
    M=2*L-1
    N=2*L
    DO 153 I=1,IP6
    DO 154 J=1,KKK
    IF(FTP(I,J,M)-FTP(11,I,1)) 154,154,155
155 FTP(I,J,N)=FTP(I,J,N)+1.0
    GO TO 153
154 CONTINUE
    FTP(I,KA,N)=FTP(I,KA,N)+1.0
153 CONTINUE
575 CONTINUE
592 IF(NSAMP-KOUNT) 157,157,100
157 TEMP1=NSAMP+1
    DO 158 I=1,IP6
    TEMP2=1.0
    TEMP3=1.0
    PRINT 159
159 FORMAT(1H1,10X,3HFPP,6X,7HFTHEORY,7X,6HFCOUNT,9X,4HFOBS,9X,4HFPCT,
   15X,8HFTHEORYA,6X,7HFCOUNTA,8X,5HFOBSA,8X,5HFPCTA)
    DO 562 J=1,KKK
    FTP(11,J,6)=FTP(I,J,6)/TEMP1
    FTP(11,J,4)=FTP(I,J,4)/TEMP1
    IF(J-1) 561,560,561
560 TEMP2=1.0
    TEMP3=1.0
    GO TO 160
561 L=J-1
    TEMP2=TEMP2-FTP(11,L,6)
    TEMP3=TEMP3-FTP(11,L,4)
160 PRINT 161,FPP(J),FTP(I,J,5),FTP(I,J,6),FTP(11,J,6),TEMP2,
   1FTP(I,J,3),FTP(I,J,4),FTP(11,J,4),TEMP3
562 CONTINUE
161 FORMAT(1X,9F13.3)
```

```
      FTP(11,KA,6)=FTP(I,KA,6)/TEMP1
      FTP(11,KA,4)=FTP(I,KA,4)/TEMP1
      TEMP2=TEMP2-FTP(I,KKK,6)/TEMP1
      TEMP3=TEMP3-FTP(I,KKK,4)/TEMP1
      PRINT 165, FTP(I,KA,6),FTP(11,KA,6),TEMP2,FTP(I,KA,4),
     1FTP(11,KA,4),TEMP3
  165 FORMAT(27X,3F13.3,13X,3F13.3)
      PRINT 580
  580 FORMAT(1H0,10X,3HFPP,6X,7HFTHEORY,7X,6HFCOUNT,9X,4HFOBS,9X,4HFPCT)
      DO 581 IA=1,KKK
      FTP(11,IA,2)=FTP(I,IA,2)/TEMP1
      IF(IA-1)582,583,582
  583 TEMP2=1.0
      GO TO 584
  582 L=IA-1
      TEMP2=TEMP2-FTP(11,L,2)
  584 PRINT 585,FPP(IA),FTP(I,IA,1),FTP(I,IA,2),FTP(11,IA,2),TEMP2
  581 CONTINUE
  585 FORMAT(1X,5F13.3)
      FTP(11,KA,2)=FTP(I,KA,2)/TEMP1
      TEMP2=TEMP2-FTP(I,KKK,2)/TEMP1
      PRINT 586,FTP(I,KA,2),FTP(11,KA,2),TEMP2
  586 FORMAT(27X,3F13.3)
  158 CONTINUE
      PRINT 593,EPS
  593 FORMAT(8H1EPSILON,F10.4)
      DO 594 I=1,14
      FI=I
      FI=0.05*FI+0.25
  594 PRINT 595,FI,EP(I)
  595 FORMAT(1X,2F13.4)
      EPSI=((TEMP1-1.0)*(FTP(11,18,2))-FTP(11,17,2)**2)/((TEMP1-1.0)*(TE
     1MP1-2.0))
      EPSM=FTP(11,17,2)/(TEMP1-1.0)
      PRINT 596,EPSM,EPSI
  596 FORMAT(1X,2E20.8)
  100 CONTINUE
      RETURN
      END
      SUBROUTINE FDISTR(MM,NN,F,P)
      A=MM
      A=A/2.0
      B=NN
      B=B/2.0
      X=XVSBTAF(A,B,P)
      F=B*X/((1.0-X)*A)
      END
      SUBROUTINE INCBR(A,B,X,P,XLB)
      XLB=XLGF(A)+XLGF(B)-XLGF(A+B)
      XI=1.0-X
      XH=X-0.5
      SA=A-1.0
      SB=B-1.0
      SAB=A-B
      I=0
      IF(X) 12,14,1
    1 IF(XI) 13,14,2
    2 IF(SA) 3,4,5
    3 IF(SB) 6,16,8
  4 IF(SB) 15,14,15
  5 IF(SB) 9,16,10
  6 IF(XH) 22,7,23
  7 IF(SAB) 22,14,23
  8 IF(XH) 41,23,24
  9 IF(XH) 21,22,44
 10 IF(XH) 21,11,24
 11 IF(SAB) 24,14,21
 12 P=0.0
    RETURN
 13 P=1.0
    RETURN
 14 P=X
    RETURN
 15 P=1.0-XI**B
    RETURN
 16 P=X**A
    RETURN
 21 P=BETA1F(A,B,X,XLB+LOGF(A))
    RETURN
 44 I=I+1
 22 R=EXPF(A*LOGF(X)-XLB-LOGF(A))
    IF(I) 45,46,45
 45 IF(ABSF(R)-0.00000001) 46,46,24
 46 P=BETA2F(A,B,X,R)
    RETURN
 41 I=I+1
 23 R=EXPF(B*LOGF(XI)-XLB-LOGF(B))
    IF(I) 42,43,42
 42 IF(ABSF(R)-0.00000001) 43,43,21
 43 P=1.0-BETA2F(B,A,XI,R)
    RETURN
 24 P=1.0-BETA1F(B,A,XI,XLB+LOGF(B))
    IND=4
    RETURN
    END

    FUNCTION BETA1F(S,T,X,R)
    EPS=0.0000000001
    Z=X/(1.0-X)
    ZK=0.0
    SUM=0.0
    TK=EXPF(S*LOGF(Z)-(S+T-1.0)*LOGF(1.0+Z)-R)
    I=-1
    M1=0
    M2=0
    M3=XINTF(T)
    TMP=(T-S)/2.0
    IF(TMP) 3,3,1
  1 M2=XINTF(TMP)
    TMP=(T*Z-S)/(1.0+Z)
    IF(TMP) 3,3,2
  2 M1=XINTF(TMP)
  3 I=I+1
    SUM=SUM+TK
    ZK=ZK+1.0
    TK=TK*Z*(T-ZK)/(S+ZK)
 12 IF(I-M3) 4,6,6
  4 IF(I-M2) 5,5,7
  5 IF(I-M1) 3,3,9
  6 IF(ABSF(TK)/(1.0+Z)-EPS) 10,10,3
  7 IF(1.0-Z) 8,11,8
 11 IF(TK) 3,10,3
  8 IF(TK/(1.0-Z)-EPS) 10,10,3
  9 TMP=M2-I
    IF((TMP+1.0/(1.0-Z))*TK-EPS) 10,10,3
 10 BETA1F=SUM+TK
    END

    FUNCTION BETA2F(S,T,X,R)
    EPS=0.0000000001
    ZK=0.0
    SUM=0.0
    TK=R
    TM=INTF(T)
    I=-1
    M3=TM
    TMP=T-TM
    IF(TMP) 2,1,2
  1 REM=0.0
    GO TO 3
  2 REM=EXPF((S+TM)*LOGF(X)+XLGF(S+T)-XLGF(S)-XLGF(TMP)-XLGF(TM+1.0))/
   1((S+TM)*(1.0-X))
  3 SUM=SUM+TK
    I=I+1
    ZK=ZK+1.0
    CK=(S+ZK-1.0)*(T-ZK)/((S+ZK)*ZK)
    SK=-X*CK
    TK=SK*TK
 15 IF(I-M3) 4,4,6
  4 IF(1.0-ABSF(CK)) 5,5,13
  5 IF(1.0-ABSF(SK)) 3,3,7
  6 IF(ABSF(TK)/(1.0-X)-EPS) 14,14,3
  7 IF (2.0*ABSF(TK)+REM-EPS) 14,14,3
 13 IF(ABSF(TK)+REM-EPS) 14,14,3
 14 BETA2F=SUM+TK
    END

    FUNCTION XLGF(X)
  1 FORMAT(11H XLGF ARG =,E16.10)
    XLGF=0.0
    Y=1.0
    Z=X
    IF(Z.GT.0.0) 2,7
  2 IF(Z.LT.10.**300.) 3,7
  3 IF(Z.GT.4.0) 9,4
  4 IF(Z.EQ.1.0) 10,5
  5 IF(Z.EQ.2.0) 10,6
  6 Y=Y*Z
    Z=Z+1.0
    IF(Z.GT.4.0) 8,6
  7 PRINT 1,Z
    STOP
  8 XLGF=LOGF(Y)
  9 Y=LOGF(Z)
    Y=Y*(Z-0.5)-Z-XLGF
    XLGF=-1.0/Z
    Z=XLGF*XLGF
    XLGF=(((((.00129175269*Z-.000847457627)*Z+.000595238095)*Z-.000793
   1650794)*Z+.002777777778)*Z-.083333333333)*XLGF+.918938533204+Y
```

```
10 END
   FUNCTION XVSBTAF(A,B,P)
   DIMENSION S(25)
   EPS=0.00000001
   SA=A-1.0
   SB=B-1.0
   SP=P
   IF(SP) 2,3,1
 1 IF(1.0-SP) 2,3,5
 2 STOP 2
 3 X=SP
 4 XVSBTAF=X
   RETURN
 5 IF(SA) 7,8,6
 6 IF(SB) 10,9,11
 7 IF(SB) 11,9,10
 8 X=1.0-(1.0-SP)**(1.0/B)
   GO TO 4
 9 X=SP**(1.0/A)
   GO TO 4
10 X=A/(A+B)
   GO TO 12
11 X=SA/(SA+SB)
12 CALL INCBR(A,B,X,PP,R)
   TP=SP-PP
   IF(ABSF(TP)-EPS) 4,4,13
13 IF(TP.LT.PP) 24,14
24 IF(PP.LT.1.0+TP) 19,14
14 XN=X+TP*EXPF(R-SA*LOGF(X)-SB*LOGF(1.0-X))
   IF(XN) 17,17,15
15 IF(1.0-XN) 18,18,16
16 X=XN
   GO TO 12
17 X=X*SP/PP
   GO TO 12
18 X=(X-X*SP+SP-PP)/(1.0-PP)
   GO TO 12
19 XL1=LOGF(X)
   XL2=LOGF(1.0-X)
   SX=X
   ABSNX=1.0
   TA=-1.0
   TB=-1.0
   TAB=0.0
   PN=1.0
   DO 20 I=1,25
20 S(I)=0.0
   S(2)=1.0
   DO 22 K=2,10
   CK=0.0
   ZK=K-1
   DO 21 I=2,K
   J=K-I+2
   ZJ=J
   S(J)=(ZJ-2.0-TA)*S(J)+SX*(TAB-ZJ+3.0)*S(J-1)
21 CK=CK+S(J)
   TA=TA+A
   TB=TB+B
   TAB=TA+TB
```

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     PN=TP*PN
     ABSOX=ABSNX
     XK=CK*PN*EXPF(ZK*R-TA*XL1-TB*XL2-XLGF(ZK+1.0))
     ABSNX=ABSF(XK)
  22 X=XK+X
     GO TO 12
     END
     MACHINE RANDEV(A)
     CON(EXP=1777B)
     CON(K1=1427075653745612B,K2=0602734560551456B,
    *K3=2136640501470023B)
     CON(NZERO=-0B)
     CON(T2=27)
     CON(T1=17)
     CON(EXP1=2002000000000000B,EXP2=2003000000000000B,
    *EXP3=2004000000000000B)
     CON(L2=2000764365300000B,L3=2000552023631477B,
    *    L4=5776233075057607B,L5=5776340724240241B,
    *    L6=1775502223671136B,L7=2002561250731211B,
    *    L8=2043000000000000B,L9=2000532710277575B,
    *    L10=6002252525252524B,L11=1775631463146631B,
    *    K12=6001377777777777B,K13=1776525252525253B,
    *    K14=5777377777777777B,K15=2000777777777777B,
    *    K16=1734400000000000B)
     CON(K0=1756400266146542B,K8=2001561250731225B,
    *    K7=7777000000000000B,K9=2000542710277575B,
    *   K10=2006520067334423B,K5=2003471632447111B,
    *    K6=1773631225674337B)
     CON(K4=1120703125)
     CON(ONE=1.0)
  1A SLJ(*)            LDA(K4)          .GET FIRST USED RZNDOM NUMBER
     MUF(SEED)         LDA(*)           .
     AJP3(2R)          ALS(1)           .
     INA(1)            MUF(K4)          .SET UP FIRST OF NEW SEQUENCE
  2R ENA(2000B)        LRS(1)           .
     STQ(SEED)         LRS(11)          .
     STQ(Y)            SLJ4(1OTHR)      .GET NEXT VARIABLE
     STQ(X)            FAD(Y)           .GET DECISION VARIABLE
     THS(.1362)        SLJ(1WAY)        .GET THE NUMBERS THE FIRST WAY
     THS(.0255)        SLJ(2WAY)        .GET THE RANDOM NUMBER THE SECOND WAY
     THS(.0026997961)    SLJ(3WAY)      .GET IT THE THIRD WAY
4WAY LDA(X)            FAD(X)           .
     FSB(ONE)          STA(X)           .
     FMU(X)            STA(X2)          .
     SLJ4(1OTHR)                        .
     LDL(NZERO)        FMU(2.0)         . DOUBLE THIS VARIABLE
     FSB(ONE)          STA(Y)           .
     FMU(Y)            STA(Y2)          .
     FAD(X2)           STA(Z)           .
     THS(ONE)          SLJ(4OUCH)       .
     LDQ(K7)                            .LOAD MASK
     THS(L2)           SLJ(1L)          .
     STA(T)            SSU(L2)          .FORM M+A
     STA(T1)           FAD(L3)          .
     STA(T2)           LDA(T1)           .FORM M-A
     FSB(L3)           FDV(T2)          .FORM T=(M-A)/(M+A)
     STA(T1)           FMU(T1)           .
     STA(T2)           FAD(L4)          .
     STA(T3)           LDA(L5)          .APPROXIMATE LOG M(BASE 2)
     FDV(T3)           FAD(L6)          .
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       FMU(T2)              FAD(L7)             .
       FMU(T1)              STA(T2)              .
       ENA(0)               LDQ(T)               .
       LLS(13)              INA(-4001B)          .
       AJP2(L+1)            INA(2)               .
       INA(-1)              SCM(L8)             .
       FAD(NZERO)           FAD(T2)             .
       FMU(L9)              SLJ(3EXIT)          .EXIT FROM LOG ROUTINE
    1L THS(ONE)             SLJ(3C)             .
       FSB(K15)             FSB(K16)             .
2C     STA(T)               ENI(0)               .
       FMU(L10)             FAD(L11)            . START 6 TERM FUNCTION IN U
       FMU(T)               FAD(K12)             .
       FMU(T)               FAD(K13)             .
       FMU(T)               FAD(K14)             .
       FMU(T)               FAD(ONE)            .
       FMU(T)               SLJ(3EXIT)          .SIX TERM FUNCTION EXIT
3C     FSB(1.0)             SLJ(2C)              .
3EXIT  FMU(-2.0)            FAD(9.0)            .
       FDV(Z)               LRS(48)             .
       ENA(0)               SIU1(1EXIT)         .SAVE B1 FOR EXIT FROM SQRTF
       ENI1(0)              LLS(2)              .
       AJP(3BB)             LLS(9)              .SENSE NEGATIVE EXPONENT
       INA(1000B)           QJP2(L+2)            .SENSE EVEN OR ODD
       ENI1(1)              INA(1)              .PROCESS ODD EXPONENT
2BB    QLS(1)               SAU(1C)             .
       ENA(0)               LRS1(3)             .ADJUST COEFICIENT
       LDL(NZERO)           QRS(2)              .FORM INITIAL ESTIMATE
       STQ(T1)              ADD(K1)              .
       STA(T2)              ENQ(-0)             .
       LAC(K2)              DVF(T2)              .
       ADD(K3)              STA(T2)              .DO TWO NEWTON ITERATIONS
       LDA(T1)              DVF(T2)             .
       ADD(T2)              ARS(1)              .
       STA(T2)              LDA(T1)             .
       DVF(T2)              ADD(T2)             .
       SCL(EXP)             ALS(2)              .PACK EXPONENT INTO RESULT
1C     INA(*)               ALS(36)             .
1EXIT  ENI1(*)              SLJ(L+5)
3BB    LLS(9)               INA(1000B)          .EXPONENT NEGATIVE
       QJP3(2BB)            ENI1(1)             .
       EQS(EXP)             SLJ(2BB)            .TEST IF EXPONENT 1777
       ENA(2000B)           SLJ(2BB)            .
       STA(Z)               FMU(X)              .FOR ONE VARIABLE
       THS(3.0)             SLJ(1OUT)           .IF IT IS BIG ENOUGH EXIT
       AJP2(L+2)            SCM(-0B)            .COMPLEMENT IF NEGATIVE
       THS(3.0)             SLJ(2OUT)           .IF BIG NEGATIVE EXIT
       LDA(Z)               FMU(Y)              .FORM SECOND TRAIL
       THS(3.0)             SLJ(1OUT)           .TEST FOR LARGE POSITIVE
       AJP2(4WAY)           SCM(-0B)            .COMPLEMENT IF NEGATIVE
       THS(3.0 )            SLJ(2OUT)           .EXIT FOR BIG NEGATIVE
4OUCH  SLJ4(1OTHR)                              .
       STQ(X)               SLJ(4WAY)           .
  1WAY SIU1(1REST)          LDA(K4)             .SAVE INDEX AND GET THE SECOND UNIVOR
       MUF(SEED)            ENA(0)              .VARIATE
       LRS(1)               LDL(NZERO)          .GET NEXT SEED INTO A AFTER MAKING PO
       QRS(2)               STQ(Y)              .SAVE SECOND UNIFORM VARIABTE
       MUF(K4)              ENA(0)              .GET SECOND UNIFORM VARIATE
       LRS(1)               STQ(SEED)           .SAVE SEEED FOR NEXT TIME
       LDA(X)               ALS(9)              .CORRECT FIRST RANDOM DEVIATE
       QRS(2)              ADL(NZERO)          .ADD TWO OF THEDEVIATES
       ADD(Y)              SCA1(2)             .SCALE IN PREPARTION FOR FLOATING
       ARS(12)             ADD1(EXP1)          .PUT IN PROPER EXPONENT
 1REST ENI1(*)             FSB(3.0)            .RESTORE INDICES AND EXIT
       SLJ(1A)                                 .
  2WAY SLJ4(10THR)                             .
       LDL(NZERO)          FAD(X)              .ADD IN THE SECOND RANDOM VARIABLE
       FSB(ONE)            FMU(1.5)            .
       SLJ(1A)             .                   GET OUT
  3WAY LDA(X)              FMU(6.)             .
       FSB(3.)             STA(X)              .
       AJP(L+1)            SCM(-0B)            .
       THS(3.0)            SLJ(3CONT)          .
       ENA(0)              SLJ(3DONE)          .
 3CONT STA(AX)             FMU(AX)             .
       STA(X2)             FMU(-0.5)           .
       STA(Z)              SIU1(1EXIT)         .SAVE B1 AND ARGUMENT
       ENI1(10)            AJP2(L+2)           .TEST FOR NEGATIVE ARGUMENT
       SCM(NZERO)          ENI1(0)             .CORRECT NEGATIVE ARGUMENT
       THS(K0)             SLJ(L+3)             .SENSE X LESS THAN A
       LDA(ONE)            FAD(Z)              .FORM ANSWER
       SLJ(2EXIT)                              .EXIT FROM ROUTINE
       FMU(K8)             STA(T)              .FORM T=ABSX/LN2
       THS(0.5)            SLJ(L+3)             .SENSE T LESS THAN 1/2
       LDQ(ONE)            STQ (T2)            .
       SLJ(1E)                                 .INTEGER PART OF Z=0
       FAD(0.5)            STA(T3)              .Z=T+1/2
       ENQ(0)              ALS(2)               .FIND INTEGER PART OF Z
       LLS(10)             STQ(T4)              .
       LDQ(K7)             QRS7(T4)            .
       LDL(T3)             STA(T1)              .
       SCL(K7)             ALS7(T4)            .
       ADD(ONE)            STA(T2)             .FORM 2 TO THE N
       LDA(T)              FSB(T1)              .
    1F FMU(K9)             STA(T)              .
       FMU(T)              STA(T1)              .
       FAD(K10)            STA(T3)             .START APPROXIMATION FOR E TO U
       LDA(K5)             FDV(T3)              .
       FAD(K6)             FMU(T1)              .
       FAD(2.0)            STA(T1)              .
       FSB(T)              STA(T3)              .
       LDA(T1)             FAD(T)               .
       FDV(T3)             FMU(T2)              .FORM 2 TO N)(E TO X)
       IJP1(L+2)           STA(T)              .WAS ORIGINAL X POSITIVE
       LDA(1.0)            FDV(T)              .FORM 1/T
 2EXIT ENI1(*)             FMU(17.49831196).
       STA(Z)              LDA(AX)             .
       FSB(1.5)            FMU(2.15787544) .
       STA(T)              LDA(AX)             .
       THS(ONE)            SLJ(3FIRS)          .
       FSB(3.0)            STA(T1)             .
       FMU(T1)             FMU(2.36785163)  .
       FAD(Z)              STA(Z)              .
       LDA(AX)                                 .
       THS(1.5)            SLJ(3DONE)          .
       FAD(T)              SLJ(3DONE)          .
 3FIRS FMU(AX)             FSB(3.0)            .
       FMU(4.73570326)     FAD(T)              .
       FAD(Z)              SLJ(3DONE)           .
 3DONE STA(Y)              SLJ4(10THR)         .
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         LDL(NZERO)          FMU(.358)          .
         THS(Y)              SLJ(30K)           .
         STA(X)              SLJ(3WAY)          .
     30K LDA(X)              SLJ(1OUT)          .
    1OUT SLJ(1A)
    2OUT SCM(-0B)            SLJ(1OUT)          .NEGATIVE NUMBER EXIT
   1OTHR SLJ(*)              LDA(K4)            .GET ANOTHER UNIFORM RANDOM VARIBABLE
         MUF(SEED)           ENA(2000B)         .
         LRS(1)              STQ (SEED)         .
         LRS(11)             SLJ(1OTHR)         .
         END
```